## **Core Operational Semantics of Proto**

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#### Programming Languages @ ACM SAC'11





# **Challenge: Spatial Computers**



**Robot Swarms** 



**Biological Computing** 



Sensor Networks





Cells during Morphogenesis



**Modular Robotics** 

# **Amorphous Medium**



Continuous space & timeInfinite number of devicesSee neighbors' past state



Approximate with:Discrete network of devicesSignals transmit state

# Proto



[Beal & Bachrach, '06]<sup>4</sup>



# **Challenge: Operational Semantics**

- Proto runs on multiple discrete platforms
  - Different communication systems
  - Different execution threading
  - Different time, space, density scales

How can we prove implementation correctness?

# **Configuration & Platform Model**



## Syntax & Evaluation Model

 $e ::= v | x | f | (f \overline{e})$ Expressions Values (num. or fields)  $v ::= n \mid \phi$ Functions f ::= Math operators 0 Sensing sense Stateful iteration rep  $if^{\overline{\overline{x}};\overline{x}'}$ Selection mux  $nbr^i \mid nbr-range^i$ Field creation Neighbourhood folding fold-hood\* Defined function F o ::= + |-| \* |/| >= |>| == |! = Math operatorsd ::=  $(def F(\overline{x}) e)$  Function definition E ::= [] | (o E e) | (o v E)(sense E) | (rep x E e) | (if<sup> $\overline{x}$ ; $\overline{x}'$ </sup> E e e) (mux e E e) | (mux e n E) | (mux E n n) (fold-hood\* f E e) | (fold-hood\* f n E)  $(nbr^{i} E) | (F \overline{n} E \overline{e})$ 

#### **Operational Semantics: Language**

$\frac{\langle \Sigma \rangle \mathbf{e} \to \langle \Sigma' \rangle \mathbf{e}'}{\langle \Sigma \rangle \mathbb{E}[\![\mathbf{e}]\!] \to \langle \Sigma' \rangle \mathbb{E}[\![\mathbf{e}']\!]}$	[CTX]
$ \begin{array}{c} ({\tt n}\in\overline{{\tt n}}\Rightarrow{\tt n}_o=1)  {\rm or}  ({\tt n}\notin\overline{{\tt n}}\Rightarrow{\tt n}_o=0) \\ \hline \langle \{sns(\overline{{\tt n}})\}\otimes\Sigma\rangle ({\tt sense \ {\tt n}}) \rightarrow \langle \{sns(\overline{{\tt n}})\}\otimes\Sigma\rangle{\tt n}_o \end{array} \end{array} $	[SNS]
$\frac{-}{\langle \mathtt{x} := \mathtt{n} \mid \Sigma \rangle \mathtt{x} \rightarrow \langle \mathtt{x} := \mathtt{n} \mid \Sigma \rangle \mathtt{n}}$	[VAR]
$\overset{-}{\langle \{\mathtt{x} := \bullet\} \otimes \Sigma \rangle (\texttt{rep x n e}) \rightarrow \langle \mathtt{x} := \mathtt{n} \mid \Sigma \rangle \mathtt{n}}$	[REP-I]
$\frac{\langle \{ x := n \} \otimes \Sigma \rangle e \to \langle \{ x := n \} \otimes \Sigma' \rangle e'}{\langle \{ x := n \} \otimes \Sigma \rangle (\operatorname{rep} x n' e) \to \langle \{ x := n \} \otimes \Sigma' \rangle (\operatorname{rep} x n' e')}$	[REP-E]
$\overbrace{\langle \{ \mathtt{x} := \mathtt{n} \} \otimes \Sigma \rangle (\texttt{rep } \mathtt{x} \ \mathtt{n}' \ \mathtt{n}'') \to \langle \mathtt{x} := \mathtt{n}'' \mid \Sigma \rangle \mathtt{n}''}^{-}$	[REP-F]
$ \begin{array}{c} (\mathtt{n}=\mathtt{1}\Rightarrow\mathtt{e}_{o}=\mathtt{e},\overline{\mathtt{x}}_{o}=\overline{\mathtt{x}}')  \mathrm{or}  (\mathtt{n}\neq\mathtt{1}\Rightarrow\mathtt{e}_{o}=\mathtt{e}',\overline{\mathtt{x}}_{o}=\overline{\mathtt{x}}) \\ \\ \langle\{\overline{\mathtt{x}}_{o}:=\overline{\mathtt{n}}\}\otimes\Sigma\rangle(\mathtt{if}^{\overline{\mathtt{x}};\overline{\mathtt{x}}'} \; \mathtt{n}\; \mathtt{e}\; \mathtt{e}') \rightarrow \langle\Sigma\rangle\mathtt{e}_{o} \end{array} $	[IF]

$$\frac{(\mathbf{n} = \mathbf{1} \Rightarrow \mathbf{n}_o = \mathbf{n}') \quad \text{or} \quad (\mathbf{n} \neq \mathbf{1} \Rightarrow \mathbf{n}_o = \mathbf{n}'')}{\langle \Sigma \rangle (\max \mathbf{n} \ \mathbf{n}' \ \mathbf{n}'') \rightarrow \langle \Sigma \rangle \mathbf{n}_o}$$
[MUX]

$$\langle \{\overline{a}: i \mapsto \overline{\mathbf{n}}\} \otimes \Sigma \rangle (\mathbf{nbr}^i \ \mathbf{n}) \to \langle \{\overline{a}: i \mapsto \overline{\mathbf{n}}\} \otimes \Sigma \mid i \mapsto \mathbf{n} \rangle \{\overline{a} \mapsto \overline{\mathbf{n}}\}$$
[NBR]

$$\overline{\langle\{\overline{a}:0\mapsto\overline{\mathbf{n}}\}\otimes\{\overline{a}:i\mapsto0\}\otimes\Sigma\rangle(\mathtt{nbr-range}^{i})\to\langle\{\overline{a}:0\mapsto\overline{\mathbf{n}}\}\otimes\{\overline{a}:i\mapsto0\}\otimes\Sigma\mid i\mapsto0\rangle\{\overline{a}\mapsto\overline{\mathbf{n}}\}} \quad [\mathrm{NBR-RNG}]$$

$$\frac{-}{\langle \Sigma \rangle (\texttt{fold-hood* f } \texttt{n}_0 \ \{a \mapsto \texttt{n}\} \otimes \phi) \to \langle \Sigma \rangle (\texttt{fold-hood* f } (\texttt{f } \texttt{n}_0 \ \texttt{n}) \ \phi)}$$
[FHOOD-R]

$$\frac{-}{\langle \Sigma \rangle (\texttt{fold-hood* f } n_0 \ 0) \to \langle \Sigma \rangle n_0}$$
[FHOOD-F]

$$\frac{\mathbf{v}_{1} \circ \mathbf{v}_{2} = \mathbf{v}_{0}}{\langle \Sigma \rangle (\circ \mathbf{v}_{1} \mathbf{v}_{2}) \rightarrow \langle \Sigma \rangle \mathbf{v}_{0}} \qquad [MATH]$$

$$\frac{(\text{def F} (\overline{\mathbf{x}}) \mathbf{e}) \in \mathcal{D}}{\langle \Sigma \rangle (\mathbf{F} \ \overline{\mathbf{v}}) \rightarrow \langle \Sigma \rangle \mathbf{e}[\overline{\mathbf{v}}/\overline{\mathbf{x}}]} \qquad [DEF]$$

#### **Operational Semantics: Platform**

$$\frac{\langle \Sigma \rangle \mathbf{e} \to \langle \Sigma' \rangle \mathbf{e}'}{N \mid a :: \langle \Sigma \rangle \mathbf{e} \to N \mid a :: \langle \Sigma' \rangle \mathbf{e}'} \qquad [DEVICE]$$

$$\frac{-}{\{a \to [\overline{\mu}']\overline{a}'\} \otimes \{a \xrightarrow{\overline{\mathbf{n}_r}} \overline{a}\} \otimes N \mid a :: \langle \overline{\mu} \otimes \Sigma \rangle \mathbf{n} \to \{a \xrightarrow{\overline{\mathbf{n}_r}} \overline{a}\} \otimes N \mid a :: \langle \Sigma \rangle \mathbf{e}_p \mid \{a \to [\overline{\mu}]\overline{a}\}} \qquad [RELOAD]$$

$$\frac{-}{N \mid (a \to [\overline{\mu}]a') \mid a' :: \langle \{a : \overline{\mu}'\} \otimes \Sigma \rangle \mathbf{e} \to N \mid a' :: \langle \{a : \overline{\mu}\} \otimes \Sigma \rangle \mathbf{e}} \qquad [RECEIVE]$$

$$\frac{-}{\{a \xrightarrow{\overline{\mathbf{n}_r}} \overline{a}\} \otimes N \mid a :: \langle \{\overline{a} : 0 \mapsto \overline{\mathbf{n}}\} \otimes \Sigma \rangle \mathbf{e} \mapsto \{a \xrightarrow{\overline{\mathbf{n}_r'}} \overline{a}'\} \otimes N \mid a :: \langle \{\overline{a}' : 0 \mapsto \overline{\mathbf{n}_r'}\} \otimes \Sigma \rangle \mathbf{e}} \qquad [MOVE]$$

$$\frac{-}{\{\mathbf{e} \xrightarrow{\overline{\mathbf{n}_r'}} a\} \otimes \{a \xrightarrow{\overline{\mathbf{n}_r}} \mathbf{e}\} \otimes N \mid a :: \langle \Sigma \rangle \mathbf{e} \to N} \qquad [DROP]$$

$$\frac{-}{N \mid a :: \langle \{sns(\overline{\mathbf{n}})\} \otimes \Sigma \rangle \mathbf{e} \mapsto N \mid a :: \langle \{sns(\overline{\mathbf{n}'})\} \otimes \Sigma \rangle \mathbf{e}} \qquad [SIGNAL]$$

### **Example Operation: Pointwise**

Σ





### **Example Operation: Pointwise**





#### **Example Operation: Feedback**





#### **Example Operation: Feedback**





#### **Example Operation: Feedback**





## **Example Operation: Neighborhood**

nbr



### **Example Operation: Restriction**





#### Problem: nbr/rep double-delay!

(rep x (sense 1) (min-hood (nbr x)))





Round 1



Round 2



Correct and consistent, but suboptimal.



Correct and consistent, but suboptimal.

# **Contributions and Future Work**

- Formalization of Proto discrete execution model
- Operational semantics for key space/time operators
- Discovery of fold-hood/feedback double-delay

- Future work:
  - Formalization of continuous/discrete relationship
  - Extension to all Proto operators
  - Discretization error estimation