FIELD-BASED COORDINATION WITH THE SHARE OPERATOR

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ABSTRACT. Field-based coordination has been proposed as a model for coordinating collective adaptive systems, promoting a view of distributed computations as functions manipulating data structures spread over space and evolving over time, called computational fields. The field calculus is a formal foundation for field computations, providing specific constructs for evolution (time) and neighbor interaction (space), which are handled by separate operators (called **rep** and **nbr**, respectively). This approach, however, intrinsically limits the speed of information propagation that can be achieved by their combined use. In this paper, we propose a new field-based coordination operator called **share**, which captures the space-time nature of field computations in a single operator that declaratively achieves: (i) observation of neighbors' values; (ii) reduction to a single local value; and (iii) update and converse sharing to neighbors of a local variable. We show that for an important class of self-stabilising computations, share can replace all occurrences of rep and nbr constructs. In addition to conceptual economy, use of the share operator also allows many prior field calculus algorithms to be greatly accelerated, which we validate empirically with simulations of frequently used network propagation and collection algorithms.

DOI:10.2168/LMCS-???

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Key words and phrases: Aggregate computing, field calculus, information propagation.

This work has been partially supported by Ateneo/CSP project "AP: Aggregate Programming" (http://ap-project.di.unito.it/) and by Italian PRIN 2017 project "Fluidware". This document does not contain technology or technical data controlled under either U.S. International Traffic in Arms Regulation or U.S. Export Administration Regulations.

1. INTRODUCTION

The number and density of networking computing devices distributed throughout our environment is continuing to increase rapidly. In order to manage and make effective use of such systems, there is likewise an increasing need for software engineering paradigms that simplify the engineering of resilient distributed systems. Aggregate programming [BPV15, VBD⁺18] is one such promising approach, providing a layered architecture in which programmers can describe computations in terms of resilient operations on "aggregate" data structures with values spread over space and evolving in time.

The foundation of this approach is field computation, formalized by the field calculus [VAB⁺18], a terse mathematical model of distributed computation that simultaneously describes both collective system behavior and the independent, unsynchronized actions of individual devices that will produce that collective behavior [AVD⁺19]. In this approach each construct and reusable component is a pure function from fields to fields—a field is a map from a set of space-time computational events to a set of values. In prior formulations, each primitive construct has also handled just one key aspect of computation: hence, one construct deals with time (i.e, **rep**, providing field evolution, in the form of periodic state updates) and one with space (i.e., **nbr**, handling neighbor interaction, in the form of reciprocal state sharing).

However, in recent work on the universality of the field calculus, we have identified that the combination of time evolution and neighbor interaction operators in the original field calculus induces a delay, limiting the speed of information propagation that can be achieved efficiently [ABDV18]. This limit is caused by the separation of state sharing (nbr) and state updates (rep), which means that any information received with a nbr operation has to be remembered with a rep before it can be shared onward during the next execution of the nbr operation, as illustrated in Figure 1.

In this paper, we address this limitation by extending the field calculus with the **share** construct, combining time evolution and neighbor interaction into a single new atomic coordination operator that simultaneously implements: (i) observation of neighbors' values; (ii) reduction to a single local value; and (iii) update of a local variable and sharing of the updated value with neighbors. The **share** construct thus allows the effects of information received from neighbors to be shared immediately after it is incorporated into state, rather than having to wait for the next round of computation.

The remainder of this paper formally develops and experimentally validates these concepts, expanding on a prior version $[ABD^+19]$ with an improved and extended presentation of the operators, complete formal semantics, analysis of key properties, and additional experimental validation. Following a review of the field calculus and its motivating context in Section 2, we introduce the **share** construct in detail in Section 3, along with formal semantics and analysis of the relationship of the **share**-based calculus with the prior field calculus. We then empirically validate the predicted acceleration of speed in frequently used network propagation and collection algorithms in Section 4, and conclude with a summary and discussion of future work in Section 5.

2. BACKGROUND, MOTIVATION, AND RELATED WORK

Programming collective adaptive systems is a challenge that has been recognized and addressed in a wide variety of different contexts. Despite the wide variety of goals and starting



Figure 1: Handling state sharing (**nbr**) and memory (**rep**) separately injects a delay while information "loops around" to where it can be shared (top), while combining state sharing and memory into the new **share** operator eliminates that delay (bottom).

points, however, the commonalities in underlying challenges have tended to shape the resulting aggregate programming approaches into several clusters of common approaches, as enumerated in [BDU⁺13]: (i) "device-abstraction" methods that abstract and simplify the programming of individual devices and interactions (e.g., TOTA [MZ09], Hood [WSBC04], chemical models [VPM⁺15], "paintable computing" [But02], Meld [ARGL⁺07]) or entirely abstract away the network (e.g., BSP [Val90], MapReduce [DG08], Kairos [GGG05]); (ii) spatial patterning languages that focus on geometric or topological constructs (e.g., Growing Point Language [Coo99], Origami Shape Language [Nag01], self-healing geometries [CN03, Kon03], cellular automata patterning [Yam07]); (iii) information summarization languages that focus on collection and routing of information (e.g., TinyDB [MFHH02], Cougar [YG02], TinyLime [CGG⁺05], and Regiment [NW04]); (iv) general purpose spacetime computing models (e.g., StarLisp [LMMD88], MGS [GGMP02, GMCS05], Proto [BB06], aggregate programming [BPV15]).

The field calculus $[VAB^+18, AVD^+19]$ belongs to the last of these classes, the general purpose models. Like other core calculi, such as λ -calculus [Chu32] or Featherweight Java [IPW01], the field calculus provides a minimal, mathematically tractable programming language—in this case with the goal of unifying across a broad class of aggregate programming approaches and providing a principled basis for integration and composition. Indeed, recent analysis [ABDV18] has determined that the current formulation of field calculus is space-time universal, meaning that it is able to capture every possible computation over collections of devices sending messages. Field calculus can thus serve as a unifying abstraction for programming collective adaptive systems, and results regarding field calculus have potential implications for all other works in this field. That same work establishing universality, however, also identified a key limitation of the current formulation of the field calculus, which we are addressing in this paper. In particular, the operators for time evolution and neighbor interaction in field calculus interact such that for most programs either the message size grows with the distance that information must travel or else information must travel significantly slower than the maximum potential speed. The remainder of this section provides a brief review of these key results from [ABDV18]: Section 2.1 introduces the underlying space-time computational model used by the field calculus, Section 2.2 provides a review of the field calculus itself, followed by its semantics in Section 2.3 and Section 2.4. Section 2.5 then explains and illustrates the problematic interaction between time evolution and neighbor interaction operators that will be addressed by the **share** operator in the next section.

2.1. **Space-Time Computation.** Field calculus considers a computational model in which a program P is periodically and asynchronously executed by each device δ . When an individual device performs a round of execution, that device follows these steps in order: (i) collects information from sensors, local memory, and the most recent messages from neighbors,¹ the latter in the form of a *neighboring value* map $\phi : \delta \rightarrow v$ from neighbors to values, (ii) evaluates program P with the information collected as its input, (iii) stores the results of the computation locally, as well as broadcasting it to neighbors and possibly feeding it to actuators, and (iv) sleeps until it is time for the next round of execution. Note that as execution is asynchronous, devices perform executions independently and without reference to the executions of other devices, except insofar as they use state that has arrived in messages. Messages, in turn, are assumed to be collected by some separate thread, independent of execution rounds.

If we take every such execution as an *event* ϵ , then the collection of such executions across space (i.e., across devices) and time (i.e., over multiple rounds) may be considered as the execution of a single aggregate machine with a topology based on information exchanges \rightsquigarrow . The causal relationship between events may then be formalized as defined in [ABDV18]:

Definition 2.1 (Event Structure). An event structure $\mathbf{E} = \langle E, \rightsquigarrow, \langle \rangle$ is a countable set of events E together with a neighboring relation $\rightsquigarrow \subseteq E \times E$ and a causality relation $\langle \subseteq E \times E$, such that the transitive closure of \rightsquigarrow forms the irreflexive partial order \langle and the set $\{\epsilon' \in E | \epsilon' < \epsilon\}$ is finite for all ϵ (i.e., \langle is locally finite).

Figure 2 shows an example of such an event structure, showing how these relations partition events into "causal past", "causal future", and non-ordered "concurrent" subspaces with respect to any given event. Interpreting this in terms of physical devices and message passing, a physical device is instantiated as a chain of events connected by \rightsquigarrow relations (representing evolution of state over time with the device carrying state from one event to the next), and any \rightsquigarrow relation between devices represents information exchange from the tail neighbor to the head neighbor. Notice that this is a very flexible and permissive model: there are no assumptions about synchronization, shared identifiers or clocks, or even regularity of events (though of course these things are not prohibited either).

In principle, an execution at ϵ can depend on information from any event in its past and its results can influence any event in its future. As we will see in Section 2.5, however, this is problematic for the field calculus as it has been previously defined.

¹Stale messages may expire after some timeout.



Figure 2: Example of a space-time event structure, comprising events (circles) and neighbor relations (arrows). Colors indicate causal structure with respect to the doubly-circled event (magenta), splitting events into causal past (red), causal future (cyan) and concurrent (non-ordered, in black). The numbers written within events represent a sample space-time value (cf. Def. 2.2) associated with that event structure. Figure adapted from [ABDV18].

Our aggregate constructs then manipulate space-time data values (see Figure 2) that map events to values for each event in an event structure:

Definition 2.2 (Space-Time Value). Let V be any domain of computational values and **E** be a given event structure. A space-time value $\Phi = \langle \mathbf{E}, f \rangle$ is a pair comprising the space and a function $f : E \to V$ that maps the events E of **E** to values.

We can then understand an aggregate computer as a "collective" device manipulating such space-time values, and the field calculus as a definition of operations defined both on individual events and simultaneously on aggregate computers.

2.2. Field Calculus. The field calculus is a tiny universal language for computation of space-time values. Figure 3 gives an abstract syntax for field calculus based on the presentation in [VAB⁺18] (covering a subset of the higher-order field calculus in [AVD⁺19], but including all of the issues addressed by the share construct). In this syntax, the overbar notation $\overline{\mathbf{e}}$ indicates a sequence of elements (e.g., $\overline{\mathbf{e}}$ stands for $\mathbf{e}_1, \mathbf{e}_2, \ldots, \mathbf{e}_n$), and multiple overbars are expanded together (e.g., $\overline{\delta} \mapsto \overline{\ell}$ stands for $\delta_1 \mapsto \ell_1, \delta_2 \mapsto \ell_2, \ldots, \delta_n \mapsto \ell_n$). There are four keywords in this syntax: def and if respectively correspond to the standard function definition and the branching expression constructs, while rep and nbr correspond to the two peculiar field calculus constructs that are the focus of this paper, respectively responsible for evolution of state over time and for sharing information between neighbors.

A field calculus program P is a set of function declarations \overline{F} and the main expression e. This main expression e simultaneously defines both the aggregate computation executed on

```
::= \overline{F} e
Ρ
                                                                                                                program
F
     ::= def d(\overline{x}) \{e\}
                                                                                                 function declaration
     ::= x | v | f(\overline{e}) | if(e) \{e\} \{e\} | nbr\{e\} | rep(e) \{(x) \Rightarrow e\}
                                                                                                              expression
f
     ::= d | b
                                                                                                         function name
     ::= \ell \mid \phi
                                                                                                                    value
     ::= c(\overline{\ell})
                                                                                                             local value
     ::= \overline{\delta} \mapsto \overline{\ell}
                                                                                                    neighboring value
```



the overall event structure of an aggregate computer and the local computation executed at each of the individual events therein. An expression e can be:

- A variable x, e.g. a function parameter.
- A value v, which can be of the following two kinds:
 - a *local value* ℓ , defined via data constructor **c** and arguments $\overline{\ell}$, such as a Boolean, number, string, pair, tuple, etc;
 - A neighboring (field) value ϕ that associates neighbor devices δ to local values ℓ , e.g., a map of neighbors to the distances to those neighbors.
- A function call $f(\overline{e})$ to either a *user-declared function* d (declared with the def keyword) or a *built-in function* b, such as a mathematical or logical operator, a data structure operation, or a function returning the value of a sensor.
- A branching expression $if(e_1)\{e_2\}$ else $\{e_3\}$, used to split a computation into operations on two isolated event structures, where/when e_1 evaluates to true or false: the result is the local value produced by the computation of e_2 in the former area, and the local value produced by the computation of e_3 in the latter.
- The nbr{e} construct, where e evaluates to a local value, creates a neighboring value mapping neighbors to their latest available result of evaluating e. In particular, each device δ:
 - (1) shares its value of **e** with its neighbors, and
 - (2) evaluates the expression into a neighboring value ϕ mapping each neighbor δ' of δ to the latest value that δ' has shared for **e**.

Note that within an if branch, sharing is restricted to work on device events within the subspace of the branch.

• The rep(e₁){(x) => e₂} construct, where e₁ and e₂ evaluate to local values, models state evolution over time: the value of x is initialized to e₁, then evolved at each execution by evaluating e₂ where x is the result at previous round.

Thus, for example, distance to the closest member of a set of "source" devices can be computed with the following simple function:

```
def mux(b, x, y) { if (b) {x} {y} }
def distanceTo(source) {
   rep (infinity) { (d) =>
      mux( source, 0, minHood(nbr{d}+nbrRange()) )
} }
```

7

Here, we use the def construct to define a distanceTo function that takes a Boolean source variable as input. The rep construct defines a distance estimate d that starts at infinity, then decreases in one of two ways. If the source variable is true, then the device is currently a source, and its distance to itself is zero. Otherwise, distance is estimated via the triangle inequality, taking the minimum of a neighboring value (built-in function minHood) of the distance to each neighbor (built-in function nbrRange) plus that neighbor's distance estimate nbr{d}. Function mux ensures that all its arguments are evaluated before being selected.

2.3. Device Semantics. The computation that takes place on a single device is formalized by a big-step semantics, expressed by the judgement $\delta; \Theta; \sigma \vdash \mathbf{e}_{main} \Downarrow \theta$, to be read "expression \mathbf{e}_{main} evaluates to θ on device δ with respect to environment Θ and sensor state σ ". The result of evaluation is a value-tree θ , which is an ordered tree of values that tracks the results of all evaluated subexpressions of \mathbf{e}_{main} . Such a result is made available to δ 's neighbors for their subsequent firing (including δ itself, so as to support a form of state across computation rounds). The recently-received value-trees of neighbors are then collected into a value-tree environment Θ , implemented as a map from device identifiers to value-trees (written $\overline{\delta} \mapsto \overline{\theta}$ as short for $\delta_1 \mapsto \theta_1, \ldots, \delta_n \mapsto \theta_n$). Intuitively, the outcome of the evaluation will depend on those value-trees. Figure 4 (top) defines value-trees and value-tree environments.

Example 2.3. The graphical representation of the value trees $6\langle 2 \rangle \langle 3 \rangle$ and $6\langle 2 \rangle \langle 3 \langle 7 \rangle \langle 1 \rangle \langle 1 \rangle \langle 1 \rangle \rangle$ is as follows:



In the following, for sake of readability, we sometimes write the value \mathbf{v} as short for the value-tree $\mathbf{v}\langle\rangle$. Following this convention, the value-tree $6\langle 2\langle\rangle, 3\langle\rangle\rangle$ is shortened to $6\langle 2, 3\rangle$, and the value-tree $6\langle 2\langle\rangle, 3\langle 7\langle\rangle, 4\langle\rangle, 4\langle\rangle\rangle\rangle$ is shortened to $6\langle 2, 3\langle 7, 1, 4\rangle\rangle$.

Figure 4 (bottom) defines the judgement $\delta; \Theta; \sigma \vdash \mathbf{e} \Downarrow \theta$, where: (i) δ is the identifier of the current device; (ii) Θ is the neighboring value of the value-trees produced by the most recent evaluation of (an expression corresponding to) \mathbf{e} on δ 's neighbors; (iii) \mathbf{e} is a closed run-time expression (i.e., a closed expression that may contain neighboring values); (iv) the value-tree θ represents the values computed for all the expressions encountered during the evaluation of \mathbf{e} —in particular the root of the value tree θ , denoted by $\rho(\theta)$, is the value computed for expression \mathbf{e} . The auxiliary function ρ is defined in Figure 4 (second frame).

The operational semantics rules are based on rather standard rules for functional languages, extended so as to be able to evaluate a subexpression \mathbf{e}' of \mathbf{e} with respect to the value-tree environment Θ' obtained from Θ by extracting the corresponding subtree (when present) in the value-trees in the range of Θ . This process, called *alignment*, is modelled by the auxiliary function π defined in Figure 4 (second frame). This function has two different behaviors (specified by its subscript or superscript): $\pi_i(\theta)$ extracts the *i*-th subtree of θ ; while $\pi^{\ell}(\theta)$ extracts the last subtree of θ , *if* the root of the first subtree of θ is equal to the local (boolean) value ℓ (thus implementing a filter specifically designed for the *if* construct). Auxiliary functions ρ and π apply pointwise on value-tree environments, as defined in Figure 4 (second frame).

Value-trees and value-tree environments: $\begin{array}{lll} \theta & ::= & \mathbf{v} \langle \overline{\theta} \rangle \\ \Theta & ::= & \overline{\delta} \mapsto \overline{\theta} \end{array}$ value-tree value-tree environment **Auxiliary functions:** $\rho(\mathbf{v}\langle\overline{\theta}\rangle) = \mathbf{v}$ $\begin{aligned} & \pi_i(\mathbf{v}\langle\theta_1,\ldots,\theta_n\rangle) = \theta_i & \text{if } 1 \le i \le n \\ & \pi_i(\theta) = \bullet & \text{otherwise} \end{aligned} \qquad \begin{aligned} & \pi^\ell(\mathbf{v}\langle\theta_1,\theta_2\rangle) = \theta_2 & \text{if } \rho(\theta_1) = \ell \\ & \pi^\ell(\theta) = \bullet & \text{otherwise} \end{aligned}$ For $aux \in \rho, \pi_i, \pi^\ell$: $\begin{cases} aux(\delta \mapsto \theta) &= \delta \mapsto aux(\theta) & \text{if } aux(\theta) \neq aux(\delta \mapsto \theta) \\ & aux(\delta \mapsto \theta) &= \bullet & \text{if } aux(\theta) = aux(\theta) \end{aligned}$ if $aux(\theta) \neq \bullet$ if $aux(\theta) = \bullet$ body(d) = e if def $d(\overline{x}) \{e\}$ $args(d) = \overline{x}$ if def $d(\overline{x})$ {e Syntactic shorthands: $\delta; \overline{\pi}(\Theta); \sigma \vdash \overline{\mathbf{e}} \Downarrow \overline{\theta} \text{ where } |\overline{\mathbf{e}}| = n \text{ for } \delta; \pi_1(\Theta); \sigma \vdash \mathbf{e}_1 \Downarrow \theta_1 \cdots \delta; \pi_n(\Theta); \sigma \vdash \mathbf{e}_n \Downarrow \theta_n$ where $|\overline{\theta}| = n$ for $\rho(\theta_1), \dots, \rho(\theta_n)$ where $|\overline{\mathbf{x}}| = n$ for $\mathbf{x}_1 := \rho(\theta_1) \dots \mathbf{x}_n := \rho(\theta_n)$ $\delta; \Theta; \sigma \vdash \mathbf{e} \Downarrow \theta$ $\rho(\overline{\theta})$ $\overline{\mathbf{x}} := \rho(\overline{\theta})$ Rules for expression evaluation: $\frac{[\text{E-LOC}]}{\delta;\Theta;\sigma \vdash \ell \Downarrow \ell \langle \rangle} \qquad \qquad \frac{[\text{E-FLD}] \quad \phi' = \phi|_{\mathbf{dom}(\Theta) \cup \{\delta\}}}{\delta;\Theta;\sigma \vdash \phi \Downarrow \phi' \langle \rangle}$ [E-B-APP] $\delta; \overline{\pi}(\Theta); \sigma \vdash \overline{\mathbf{e}} \Downarrow \overline{\theta} \quad \mathbf{v} = (\mathbf{b})^{\delta}_{\sigma} \Theta(\rho(\overline{\theta}))$ $\delta; \Theta; \sigma \vdash \mathbf{b}(\overline{\mathbf{e}}) \Downarrow \mathbf{v}(\overline{\theta})$ $\frac{\delta; \overline{\pi}(\Theta); \sigma \vdash \overline{\mathbf{e}} \Downarrow \overline{\theta} \qquad \delta; \Theta; \sigma \vdash body(\mathtt{d})[args(\mathtt{d}) := \rho(\overline{\theta})] \Downarrow \theta'}{\delta; \Theta; \sigma \vdash \mathtt{d}(\overline{\mathbf{e}}) \Downarrow \rho(\theta') \langle \overline{\theta}, \theta' \rangle}$ [E-D-APP] $\frac{\delta; \pi_1(\Theta); \sigma \vdash \mathbf{e} \Downarrow \theta}{\delta; \Theta; \sigma \vdash \mathbf{nbr}\{\mathbf{e}\} \Downarrow \phi \langle \theta \rangle} \phi = \rho(\pi_1(\Theta))[\delta \mapsto \rho(\theta)]$ [E-NBR] $\begin{array}{ccc} \delta; \pi_1(\Theta); \sigma \vdash \mathbf{e}_1 \Downarrow \theta_1 \\ \delta; \pi_2(\Theta); \sigma \vdash \mathbf{e}_2[\mathbf{x} := \ell_0] \Downarrow \theta_2 \end{array} \quad \ell_0 = \left\{ \begin{array}{ccc} \rho(\pi_2(\Theta))(\delta) & \text{if } \delta \in \mathbf{dom}(\Theta) \\ \rho(\theta_1) & \text{otherwise} \end{array} \\ \delta; \Theta; \sigma \vdash \mathbf{rep}(\mathbf{e}_1)\{(\mathbf{x}) \implies \mathbf{e}_2\} \Downarrow \rho(\theta_2) \langle \theta_1, \theta_2 \rangle \end{array} \right.$ [E-REP] $\begin{array}{c} \text{[E-IF] } \delta; \pi_1(\Theta); \sigma \vdash \mathbf{e} \Downarrow \theta_1 \qquad \rho(\theta_1) \in \{\texttt{true}, \texttt{false}\} \qquad \delta; \pi^{\rho(\theta_1)}(\Theta); \sigma \vdash \mathbf{e}_{\rho(\theta_1)} \Downarrow \theta \\ \hline \delta; \Theta; \sigma \vdash \texttt{if}(\mathbf{e})\{\texttt{e}_{\texttt{true}}\}\{\texttt{e}_{\texttt{false}}\} \Downarrow \rho(\theta) \langle \theta_1, \theta \rangle \end{array}$

Figure 4: Big-step operational semantics for expression evaluation.

Rules [E-LOC] and [E-FLD] model the evaluation of expressions that are either a local value or a neighboring value, respectively: note that in [E-FLD] we take care of restricting the domain of a neighboring value to the only set of neighbor devices as reported in Θ .

Rule [E-B-APP] models the application of built-in functions. It is used to evaluate expressions of the form $\mathbf{b}(\mathbf{e}_1 \cdots \mathbf{e}_n)$, where $n \geq 0$. It produces the value-tree $\mathbf{v}\langle\theta_1,\ldots,\theta_n\rangle$, where θ_1,\ldots,θ_n are the value-trees produced by the evaluation of the actual parameters $\mathbf{e}_1,\ldots,\mathbf{e}_n$ and \mathbf{v} is the value returned by the function. The rule exploits the special auxiliary function $\|\mathbf{b}\|_{\delta}^{\Theta}$, whose actual definition is abstracted away. This is such that $\|\mathbf{b}\|_{\delta}^{\Theta}(\overline{\mathbf{v}})$ computes the

result of applying built-in function **b** to values $\overline{\mathbf{v}}$ in the current environment of the device δ . In particular: the built-in 0-ary function **self** gets evaluated to the current device identifier (i.e., $(\texttt{self})^{\Theta}_{\delta}() = \delta$), and mathematical operators have their standard meaning, which is independent from δ and Θ (e.g., $(*)^{\Theta}_{\delta}(2,3) = 6$).

Example 2.4. Evaluating the expression *(2,3) produces the value-tree $6\langle 2,3 \rangle$. The value of the whole expression, 6, has been computed by using rule [E-B-APP] to evaluate the application of the multiplication operator * to the values 2 (the root of the first subtree of the value-tree) and 3 (the root of the second subtree of the value-tree).

The $(b)_{\delta}^{\Theta}$ function also encapsulates measurement variables such as nbrRange and interactions with the external world via sensors and actuators.

Rule [E-D-APP] models the application of a user-defined function. It is used to evaluate expressions of the form $d(e_1 \ldots e_n)$, where $n \ge 0$. It resembles rule [E-B-APP] while producing a value-tree with one more subtree θ' , which is produced by evaluating the body of the function d (denoted by body(d)) substituting the formal parameters of the function (denoted by args(d)) with the values obtained evaluating $e_1, \ldots e_n$.

Rule [E-REP] implements internal state evolution through computational rounds: expression $rep(e_1)\{(x) \Rightarrow e_2\}$ evaluates to $e_2[x := v]$ where v is obtained from e_1 on the first evaluation, and from the previous value of the whole rep-expression on other evaluations.

Example 2.5. To illustrate rule [E-REP], as well as computational rounds, we consider program $rep(1)\{(x) \Rightarrow *(x, 2)\}$. The first firing of a device δ is performed against the empty tree environment. Therefore, according to rule [E-REP], to evaluate $rep(1)\{(x) \Rightarrow *(x, 2)\}$ means to evaluate the subexpression *(1, 2), obtained from *(x, 2) by replacing x with 1. This produces the value-tree $\theta = 2\langle 1, 2\langle 1, 2 \rangle \rangle$, where root 2 is the overall result as usual, while its sub-trees are the result of evaluating the first and second argument respectively. Any subsequent firing of the device δ is performed with respect to a tree environment Θ that associates to δ the outcome θ of the most recent firing of δ . Therefore, evaluating $rep(1)\{(x) \Rightarrow *(x, 2)\}$ at the second firing means to evaluate the subexpression *(2, 2), obtained from *(x, 2) by replacing x with 2, which is the root of θ . Hence the results of computation are 2, 4, 8, and so on.

Rule [E-NBR] models device interaction. It first collects neighbors' values for expressions \mathbf{e} as $\phi = \rho(\pi_1(\Theta))$, then evaluates \mathbf{e} in δ and updates the corresponding entry in ϕ .

Example 2.6. To illustrate rule [E-NBR], consider $e' = \min Hood(nbr\{snsNum()\})$, where the 1ary built-in function minHood returns the lower limit of values in the range of its neighboring value argument, and the 0-ary built-in function snsNum returns the numeric value measured by a sensor. Suppose that the program runs on a network of three devices δ_A , δ_B , and δ_C where:

- δ_B and δ_A are mutually connected, δ_B and δ_C are mutually connected, while δ_A and δ_C are not connected;
- snsNum returns 1 on δ_A , 2 on δ_B , and 3 on δ_C ; and
- all devices have an initial empty tree-environment \emptyset .

Suppose that device δ_A is the first device that fires: the evaluation of $\operatorname{snsNum}()$ on δ_A yields 1 (by rules [E-LOC] and [E-B-APP], since $(\operatorname{snsNum})_{\delta_A}^{\emptyset}() = 1$); the evaluation of $\operatorname{nbr}{\operatorname{snsNum}})$ on δ_A yields ($\delta_A \mapsto 1$)(2) (by rule [E-NBR]); and the evaluation of \mathbf{e}' on δ_A yields

$$\theta_A = 1 \langle (\delta_A \mapsto 1) \langle 1 \rangle \rangle$$

(by rule [E-B-APP], since $(\min Hood)_{\delta_A}^{\emptyset}(\delta_A \mapsto 1) = 1$). Therefore, at its first fire, device δ_A produces the value-tree θ_A . Similarly, if device δ_C is the second device that fires, it produces the value-tree

$$\theta_C = 3 \langle (\delta_C \mapsto 3) \langle 3 \rangle \rangle$$

Suppose that device δ_B is the third device that fires. Then the evaluation of \mathbf{e}' on δ_B is performed with respect to the environment $\Theta_B = (\delta_A \mapsto \theta_A, \ \delta_C \mapsto \theta_C)$ and the evaluation of its subexpressions $nbr\{snsNum()\}$ and snsNum() is performed, respectively, with respect to the following value-tree environments obtained from Θ_B by alignment:

$$\begin{aligned} \Theta'_B &= \pi_1(\Theta_B) = (\delta_A \mapsto (\delta_A \mapsto 1)\langle 1 \rangle, \ \delta_C \mapsto (\delta_C \mapsto 3)\langle 3 \rangle) \\ \Theta''_B &= \pi_1(\Theta'_B) = (\delta_A \mapsto 1, \ \delta_C \mapsto 3) \end{aligned}$$

We thus have that $(snsNum)_{\delta_B}^{\Theta''_B}() = 2$; the evaluation of $nbr\{snsNum()\}$ on δ_B with respect to Θ'_B produces the value-tree $\phi\langle 2 \rangle$ where $\phi = (\delta_A \mapsto 1, \delta_B \mapsto 2, \delta_C \mapsto 3)$; and $(minHood)_{\delta_B}^{\Theta_B}(\phi) = 1$. Therefore the evaluation of \mathbf{e}' on δ_B produces the value-tree $\theta_B = 1\langle \phi\langle 2 \rangle\rangle$. Note that, if the network topology and the values of the sensors will not change, then: any subsequent fire of device δ_B will yield a value-tree with root 1 (which is the minimum of snsNum across δ_A , δ_B and δ_C); any subsequent fire of device δ_A will yield a value-tree with root 1 (which is the minimum of snsNum across δ_A and δ_B); and any subsequent fire of device δ_C will yield a value-tree with root 2 (which is the minimum of snsNum across δ_B and δ_C).

Rule [E-IF] is almost standard, except that it performs domain restriction $\pi^{true}(\Theta)$ (resp. $\pi^{false}(\Theta)$) in order to guarantee that subexpression e_{true} is not matched against value-trees obtained from e_{false} (and vice-versa).

2.4. Network Semantics. The overall network evolution is formalized by the small-step operational semantics given in Figure 5 as a transition system on network configurations N. Figure 5 (top) defines key syntactic elements to this end. Ψ models the overall status of the devices in the network at a given time, as a map from device identifiers to value-tree environments. From it, we can define the state of the field at that time by summarizing the current values held by devices. τ models network topology, namely, a directed neighboring graph, as a map from device identifiers to set of identifiers (denoted as I). Σ models sensor (distributed) state, as a map from device identifiers to (local) sensors (i.e., sensor name/value maps denoted as σ). Then, Env (a couple of topology and sensor state) models the system's environment. So, a whole network configuration N is a couple of a status field and environment.

We use the following notation for status fields. Let $\overline{\delta} \mapsto \Theta$ denote a map from device identifiers $\overline{\delta}$ to the same value-tree environment Θ . Let $\Theta_0[\Theta_1]$ denote the value-tree environment with domain $\mathbf{dom}(\Theta_0) \cup \mathbf{dom}(\Theta_1)$ coinciding with Θ_1 in the domain of Θ_1 and with Θ_0 otherwise. Let $\Psi_0[\Psi_1]$ denote the status field with the *same domain* as Ψ_0 made of $\delta \mapsto \Psi_0(\delta)[\Psi_1(\delta)]$ for all δ in the domain of $\Psi_1, \delta \mapsto \Psi_0(\delta)$ otherwise.

We consider transitions $N \xrightarrow{act} N'$ of two kinds: firings, where *act* is the corresponding device identifier, and environment changes, where *act* is the special label *env*. This is formalized in Figure 5 (bottom). Rule [N-FIR] models a computation round (firing) at device

System configurations and action labels:				
Ψ ::= $\overline{\delta} \mapsto \overline{\Theta}$	status field			
$\tau \qquad ::= \ \overline{\delta} \mapsto \overline{I}$	topology			
$\Sigma \qquad ::= \ \overline{\delta} \mapsto \overline{\sigma}$	sensors-map			
$Env ::= \tau, \Sigma$	environment			
$N ::= \langle Env; \Psi \rangle$	network configuration			
act ::= δ env	action label			
Environment well-formedness: $WFE(\tau, \Sigma)$ holds iff $\mathbf{dom}(\tau) = \mathbf{dom}(\Sigma)$ and $\tau(\delta) \subseteq \mathbf{dom}(\Sigma)$ for all $\delta \in \mathbf{dom}(\Sigma)$.				
Transition rules for network evolution: $N \xrightarrow{act} N$				
[N-FIR] $Env = \tau, \Sigma \tau(\delta) = \overline{\delta} \delta; F(\Psi)(\delta)$	$; \Sigma(\delta) \vdash \mathbf{e}_{\mathtt{main}} \Downarrow \theta \Psi_1 = \overline{\delta} \mapsto \{\delta \mapsto \theta\}$			
$\langle Env; \Psi \rangle \xrightarrow{o} \langle Env; F(\Psi)[\Psi_1] \rangle$				
[N-ENV] $WFE(Env') Env'$	$=\tau, \overline{\delta} \mapsto \overline{\sigma} \qquad \Psi_0 = \overline{\delta} \mapsto \emptyset$			
$\langle Env; \Psi \rangle \xrightarrow{env} \langle Env'; \Psi_0[\Psi] \rangle$				

Figure 5: Small-step operational semantics for network evolution.

 δ : it takes the local value-tree environment filtered out of old values $F(\Psi)(\delta)$;² then by the single device semantics it obtains the device's value-tree θ ,³ which is used to update the system configuration of δ and of δ 's neighbors.

Rule [N-ENV] takes into account the change of the environment to a new well-formed environment Env'—environment well-formedness is specified by the predicate WFE(Env)in Figure 5 (middle). Let $\overline{\delta}$ be the domain of *Env*. We first construct a status field Ψ_0 associating to all the devices of Env' the empty context \emptyset . Then, we adapt the existing status field Ψ to the new set of devices: $\Psi_0[\Psi]$ automatically handles removal of devices, map of new devices to the empty context, and retention of existing contexts in the other devices.

Example 2.7. Consider a network of devices with $e' = \min Hood(nbr\{snsNum()\})$ as introduced in Example 2.6. The network configuration illustrated at the beginning of Example 2.6 can be generated by applying rule [N-ENV] to the empty network configuration. I.e., we have

$$\langle \emptyset, \emptyset; \emptyset \rangle \xrightarrow{env} \langle Env_0; \Psi_0 \rangle$$

where

- $Env_0 = \tau_0, \Sigma_0,$
- $\tau_0 = (\delta_A \mapsto \{\delta_B\}, \delta_B \mapsto \{\delta_A, \delta_C\}, \delta_C \mapsto \{\delta_B\}),$ $\Sigma_0 = (\delta_A \mapsto (\operatorname{snsNum} \mapsto 1), \delta_B \mapsto (\operatorname{snsNum} \mapsto 2), \delta_C \mapsto (\operatorname{snsNum} \mapsto 3)), \text{ and }$

²Function $F(\Psi)$ in rule [N-FIR] models a filtering operation that clears out old stored values from the value-tree environments in Ψ , implicitly based on space/time tags.

³We shall assume that any device firing is guaranteed to terminate in any environmental condition. Termination of a device firing is clearly not decidable, but we shall assume—without loss of generality for the results of this paper—that a decidable subset of the termination fragment can be identified (e.g., by ruling out recursive user-defined functions or by applying standard static analysis techniques for termination).

• $\Psi_0 = (\delta_A \mapsto \emptyset, \delta_B \mapsto \emptyset, \delta_C \mapsto \emptyset).$

Then, the three firings of devices δ_A , δ_C and δ_B illustrated in Example 2.6 correspond to the following transitions, respectively.

(1) $\langle Env_0; \Psi_0 \rangle \xrightarrow{\delta_A} \langle Env_0; \Psi' \rangle$, where • $\Psi' = (\delta_A \mapsto (\delta_A \mapsto \theta_A), \ \delta_B \mapsto (\delta_A \mapsto \theta_A), \ \delta_C \mapsto \emptyset)$, and • $\theta_A = 1 \langle (\delta_A \mapsto 1) \langle 1 \rangle \rangle$; (2) $\langle Env_0; \Psi' \rangle \xrightarrow{\delta_C} \langle Env_0; \Psi'' \rangle$, where • $\Psi'' = (\delta_A \mapsto (\delta_A \mapsto \theta_A), \ \delta_B \mapsto (\delta_A \mapsto \theta_A, \delta_C \mapsto \theta_C), \ \delta_C \mapsto (\delta_C \mapsto \theta_C))$, and • $\theta_C = 1 \langle (\delta_C \mapsto 3) \langle 3 \rangle \rangle$; (3) $\langle Env_0; \Psi'' \rangle \xrightarrow{\delta_B} \langle Env_0; \Psi''' \rangle$, where • $\Psi''' = (\delta_A \mapsto (\delta_A \mapsto \theta_A, \delta_B \mapsto \theta_B), \ \delta_B \mapsto (\delta_A \mapsto \theta_A, \delta_B \mapsto \theta_B, \delta_C \mapsto \theta_C), \ \delta_C \mapsto (\delta_B \mapsto \theta_B, \delta_C \mapsto \theta_C)),$ • $\theta_B = 1 \langle \phi \langle 2 \rangle \rangle$, and

•
$$\phi = (\delta_A \mapsto 1, \delta_B \mapsto 2, \delta_C \mapsto 3).$$

2.5. Problematic Interaction between rep and nbr Constructs. Unfortunately, the apparently straight-forward combination of state evolution with nbr and state sharing with rep turns out to contain a hidden delay, which was identified and explained in [ABDV18]. This problem may be illustrated by attempting to construct a simple function that spreads information from an event as quickly as possible. Let us say there is a Boolean space-time value condition, and we wish to compute a space-time function ever that returns true precisely at events where condition is true and in the causal future of those events—i.e., spreading out at the maximum theoretical speed throughout the network of devices. One might expect this could be implemented as follows in field calculus:

```
def ever1(condition) {
  rep (false) { (old) => anyHoodPlusSelf(nbr{old}) || condition }
}
```

where anyHoodPlusSelf is a built-in function that returns true if any value is true in its neighboring value input (including the value old held for the current device). Walking through the evaluation of this function, however, reveals that there is a hidden delay. In each round, the old variable is updated, and will become true if either condition is true now for the current device or if old was true in the previous round for the current device or for any of its neighbors. Once old becomes true, it stays true for the rest of the computation. Notice, however, that a neighboring device does not actually learn that condition is true, but that old is true. In an event where condition first becomes true, the value of old that is shared is still false, since the rep does not update its value until after the nbr has already been evaluated. Only in the next round do neighbors see an updated value of old, meaning that ever1 is not spreading information fast enough to be a correct implementation of ever.

We might try to improve this routine by directly sharing the value of condition:

```
def ever2(condition) {
  rep (false) { (old) => anyHoodPlusSelf(nbr{old || condition}) }
}
```

This solves the problem for immediate neighbors, but does not solve the problem for neighbors of neighbors, which still have to wait an additional round before old is updated.

In fact, it appears that the only way to avoid delays at some depth of neighbor relations is by using unbounded recursion, as previously outlined in [ABDV18]:

```
def ever3(condition) {
  rep (false) { (old) =>
    if (countHood() == 0) { old || condition } {
      ever3(anyHoodPlusSelf(nbr{old || condition}))
} }
```

where countHood counts the number of neighbors, i.e., determining whether any neighbor has reached the same depth of recursion in the branch. Thus, in ever3, neighbors' values of cond are fed to a nested call to ever3 (if there are any); and this process is iterated until no more values to be considered are present. This function therefore has a recursion depth equal to the longest sequence of events $\epsilon_0 \rightsquigarrow \ldots \rightsquigarrow \epsilon$ ending in the current event ϵ , inducing a linearly increasing computational time and message size and making the routine effectively infeasible for long-running systems.

This case study illustrates the more general problem of delays induced by the interaction of **rep** and **nbr** constructs in field calculus, as identified in [ABDV18]. With these constructs, it is never possible to build computations involving long-range communication that are as fast as possible and also lightweight in the amount of communication required.

3. The Share Construct

In order to overcome the problematic interaction between **rep** and **nbr**, we propose a new construct that combines aspects of both:

$$\mathtt{share}(\mathtt{e}_1)\{(\mathtt{x}) \Rightarrow \mathtt{e}_2\}$$

While the syntax of this new **share** construct is identical to that of **rep**, the two constructs differ in the way the construct variable **x** is interpreted each round:

- in rep, the value of x is the local value produced by evaluating the construct in the previous round, or the result of evaluating e₁ if there is no prior-round value;
- in share, on the other hand, \mathbf{x} is a *neighboring value* comprising that same value for the current device plus the values of the construct produced by neighbors in their most recent evaluation. In other words, \mathbf{x} is substituted with the evaluation of the share construct in all neighbouring events, using the local evaluation of \mathbf{e}_1 for the current device if a past evaluation is not available.

Notice that since \mathbf{x} is a neighboring value rather than a local value, \mathbf{e}_2 is responsible for processing it into a local value that can be shared with neighbors at the end of the evaluation. These apparently small differences in the construct interpretations have significant effects, allowing **share** to improve the dynamics of many algorithms.

Section 3.1 presents the operational semantics of the **share** construct. Section 3.2 introduces automatic rewritings of **rep** constructs into **share** constructs: two preserving the behavior, thus showing that **share** has the expressive power to substitute most usages of **rep** and **nbr** in programs; and one changing the behavior (in fact, improving it in many cases). Section 3.3 demonstrates the automatic behavior improvement for the example in Section 2.5, while estimating the general communication speed improvement induced by the

Auxiliary functions:				
$\phi_0[\phi_1] = \phi_2$	where $\phi_2(\delta) = \begin{cases} \phi_1(\delta) & \text{if } \delta \in \mathbf{dom}(\phi_1) \\ \phi_0(\delta) & \text{otherwise} \end{cases}$			
Rule for expression evaluation:				
	$\delta; \pi_1(\Theta); \sigma \vdash \mathbf{e}_1 \Downarrow \theta_1 \qquad \qquad \phi' = \rho(\pi_2(\Theta)) \qquad \qquad \phi = (\delta \mapsto \rho(\theta_1))[\phi']$			
[E-SHARE]	$\delta; \pi_2(\Theta); \sigma \vdash e_2[x := \phi] \Downarrow \theta_2$			
$\delta; \Theta; \sigma \vdash \mathtt{share}(\mathtt{e}_1)\{(\mathtt{x}) \Rightarrow \mathtt{e}_2\} \Downarrow \rho(\theta_2) \langle \theta_1, \theta_2 \rangle$				

Figure 6: Operational semantics for the share construct.

rewriting. Section 3.4 shows examples for which the rewriting fails to preserve the intended behavior, and Section 3.5 concludes by showing that behavior is preserved for the relevant subset of field calculus pinpointed in [VAB⁺18].

3.1. **Operational Semantics.** Formal operational semantics for the **share** construct is presented in Figure 6 (bottom frame), as an extension to the semantics given in Section 2.3. The evaluation rule is based on the auxiliary functions given in Figure 6 (top frame), plus the auxiliary functions in Figure 4 (second frame). In particular, we use the notation $\phi_0[\phi_1]$ to represent "field update", so that its result ϕ_2 has $\mathbf{dom}(\phi_2) = \mathbf{dom}(\phi_0) \cup \mathbf{dom}(\phi_1)$ and coincides with ϕ_1 on its domain, or with ϕ_0 otherwise.

The evaluation rule [E-SHARE] produces a value-tree with two branches (for \mathbf{e}_1 and \mathbf{e}_2 respectively). First, it evaluates \mathbf{e}_1 with respect to the corresponding branches of neighbors $\pi_1(\Theta)$ obtaining θ_1 . Then, it collects the results for the construct from neighbors into the neighboring value $\phi' = \rho(\pi_2(\Theta))$. In case ϕ' does not have an entry for δ , $\rho(\theta_1)$ is used obtaining $\phi = (\delta \mapsto \rho(\theta_1))[\phi']$. Finally, ϕ is substituted for \mathbf{x} in the evaluation of \mathbf{e}_2 (with respect to the corresponding branches of neighbors $\pi_2(\Theta)$) obtaining θ_2 , setting $\rho(\theta_2)$ to be the overall value.

Example 3.1 (Operational Semantics). Consider the body of function ever:

```
def ever(condition) {
    share (false) { (old) => anyHoodPlusSelf(old) || condition }
}
```

Suppose that device $\delta = 0$ first executes a round of computation without neighbors (i.e., Θ is empty), and with condition equal to false. The evaluation of the share construct proceeds by evaluating false into $\theta_1 = \text{false}\langle\rangle$, gathering neighbor values into $\phi' = \bullet$ (no values are present), and adding the value for the current device obtaining $\phi = (0 \mapsto \text{false})[\bullet] = 0 \mapsto \text{false}$. Finally, the evaluation completes by storing in θ_2 the result of anyHoodPlusSelf($0 \mapsto \text{false}$)||false (which is false $\langle \ldots \rangle^4$). At the end of the round, device 0 sends a broadcast message containing the result of its overall evaluation, and thus including $\theta^0 = \text{false}\langle \text{false}, \text{false} \langle \ldots \rangle \rangle$.

Suppose now that device $\delta = 1$ receives the broadcast message and then executes a round of computation where condition is true. The evaluation of the share constructs starts similarly as before with $\theta_1 = \text{false}\langle\rangle, \phi' = 0 \mapsto \text{false}, \phi = 0 \mapsto \text{false}, 1 \mapsto$

 $^{^{4}}$ We omit the part of the value tree that are produced by semantic rules not included in this paper, and refer to [VAB⁺18, Electronic Appendix] for the missing parts.

false. Then the body of the share is evaluated as anyHoodPlusSelf($0 \mapsto \texttt{false}, 1 \mapsto \texttt{false}$)||true into θ_2 , which is true(...). At the end of the round, device 1 broadcasts the result of its overall evaluation, including $\theta^1 = \texttt{true}(\texttt{false}, \texttt{true}(...))$.

Then, suppose that device $\delta = 0$ receives the broadcast from device 1 and then performs another round of computation with condition equal to false. As before, $\theta_1 = \texttt{false}\langle\rangle, \phi = \phi' = 0 \mapsto \texttt{false}, 1 \mapsto \texttt{true}$ and the body is evaluated as anyHoodPlusSelf($0 \mapsto \texttt{false}, 1 \mapsto \texttt{true}$)||false which produces $\texttt{true}\langle\ldots\rangle$ for an overall result of $\theta^2 = \texttt{true}\langle\texttt{false},\texttt{true}\langle\ldots\rangle\rangle$.

Finally, suppose that device $\delta = 1$ does not receive that broadcast and discards 0 from its list of neighbor before performing another round of computation with condition equal to false. Then, $\theta_1 = \texttt{false}\langle\rangle, \phi' = 1 \mapsto \texttt{true}, \phi = (1 \mapsto \texttt{false})[1 \mapsto \texttt{true}] = 1 \mapsto \texttt{true}$, and the body is evaluated as anyHoodPlusSelf $(1 \mapsto \texttt{true}) | |\texttt{false}$ which produces $\texttt{true}\langle \ldots \rangle$.

3.2. Automatic Rewritings of rep Constructs into share Constructs. The share construct can be automatically incorporated into programs using rep and nbr in few ways. First, we may want to rewrite a program while maintaining the behavior unchanged, thus showing that the expressive power of share is enough to replace other constructs to some extent. In particular, we can fully replace the rep construct through the following rewriting, expressed through the notation $\mathbf{e}[\mathbf{e}_1 := \mathbf{e}'_1, \ldots, \mathbf{e}_n := \mathbf{e}'_n]$ representing an expression \mathbf{e} in which the distinct subexpressions $\mathbf{e}_1, \ldots, \mathbf{e}_n$ have been simultaneously replaced by the corresponding expressions $\mathbf{e}'_1, \ldots, \mathbf{e}'_n$ —if \mathbf{e}_i is a subexpression of \mathbf{e}_j (for some $i \neq j$) then the occurrences \mathbf{e}_j are replaced by \mathbf{e}'_j .

Rewriting 1 (rep-elimination).

 $rep(e_1)\{(x) \Rightarrow e_2\} \longrightarrow share(e_1)\{(x) \Rightarrow e_2[x := localHood(x)]\}$

where localHood is a built-in operator that given a neighboring value ϕ returns the local value $\phi(\delta)$ for the current device.

Theorem 3.2. Rewriting 1 preserves the program behavior.

Proof. Correctness follows since the value $\phi(\delta)$ in the neighbouring value ϕ substituted for **x** in the **share** construct corresponds exactly to the value that is substituted for **x** in the corresponding **rep** construct.

In addition to eliminating **rep** occurrences, the **share** construct is able to factor out many common usages of the **nbr** construct as well (even though not all of them), as per the following equivalent rewriting. For ease of presentation, we extend the syntax of share to handling multiple input-output values: $share(e_1, e_2)\{(x_1, x_2) \Rightarrow e'_1, e'_2\}$, to be interpreted as a shorthand for a single-argument construct where the multiple input-output values have been gathered into a tuple (unpacking them before computing e'_1, e'_2 and then packing their result).

Rewriting 2 (nbr-elimination).

$$\begin{split} \texttt{rep}(\texttt{e}_1)\{(\texttt{x}) \texttt{=>} \texttt{e}_2\} &\longrightarrow \\ \texttt{fst}(\texttt{share}(\texttt{e}_1,\texttt{e}_1)\{(\texttt{x},\texttt{y})\texttt{=>} \\ & \texttt{e}_2[\texttt{y}:\texttt{=localHood}(\texttt{x}),\texttt{nbr}\{\texttt{x}\}:\texttt{=localChange}(\texttt{y},\texttt{localHood}(\texttt{x}))], \\ & \texttt{localHood}(\texttt{x}) \\ & \texttt{}) \end{split}$$

where **y** is fresh variable and localChange (ϕ, ℓ) updates the value of ϕ for the current device δ with ℓ , returning $\phi[\delta \mapsto \ell]$.

Theorem 3.3. Rewriting 2 preserves the program behavior.

Proof. We prove by induction that the two components of the **share** translation correspond to the **rep** current and previous results (respectively, using e_1 if no such previous value is available). On initial rounds of evaluation, the **share** construct evaluates to $e_2[x := e_1, nbr{x} := nbr{e_1}], e_1$ (by substituting x, y by e_1), as the **rep** construct. On other rounds, the second component of **share** is localHood(x), which is the previous result of the first component of **share**, which is the previous result of the **rep** construct by inductive hypothesis. Furthermore, the first component of **share** is e_2 with arguments localHood(x) (again, the previous result of the **rep** construct) and localChange(y, localHood(x)), which is the neighbours' values for the second argument together with the previous value of the **rep** construct for the current device. On the other hand, $nbr{x}$ is the neighbours' values for the rep construct, together with the local previous value of the **rep** construct. By inductive hypothesis, the two things coincide, concluding the proof.

However, a more interesting rewriting is the following *non-equivalent* one, which for many algorithms is able to automatically improve the communication speed while preserving the overall meaning.

Rewriting 3 (non-equivalent).

 $rep(e_1)\{(x) \Rightarrow e_2\} \longrightarrow share(e_1)\{(x) \Rightarrow e_2[x := localHood(x), nbr\{x\} := x]\}$

In particular, we shall see in Section 3.3 how this rewriting translates the inefficient ever1 routine into a program equivalent to ever3, and in Section 3.5 that this rewriting preserves the eventual behavior of a whole fragment of field calculus programs, while improving its efficiency. In particular, the improvement in communication speed can be estimated to be at least three-fold (see Section 3.3). Unfortunately, programs may exist for which this translation fails to preserve the intended meaning (see Section 3.4). This usually happens for time-based algorithms where the one-round delay is incorporated into the logic of the algorithm, or weakly characterised functions which may need reduced responsiveness for allowing results to stabilise. Thus, better performing alternatives using **share** may still exist after the program logic has been accordingly fixed.

3.3. The share Construct Improves Communication Speed. To illustrate how share solves the problem illustrated in Section 2.5, let us once again consider the ever function discussed in that section, for propagating when a condition Boolean has ever become true. By applying Rewriting 3 to the ever1 function introduced in Section 2.5 we obtain exactly the ever function introduced in Section 3.1:

```
def ever(condition) {
   share (false) { (old) => anyHoodPlusSelf(old) || condition }
}
```

Function ever is simultaneously (i) compact and readable, even more so than ever1 and ever2 (note that we no longer need to include the nbr construct); (ii) lightweight, as it involves the communication of a single Boolean value each round and few operations; and (iii) optimally efficient in communication speed, since it is true for any event ϵ with a causal predecessor $\epsilon' \leq \epsilon$ where condition was true. In particular

- in such an event ε' the overall share construct is true, since it goes to anyHoodPlusSelf(old) || true regardless of the values in old;
- in any subsequent event ϵ'' (i.e. $\epsilon' \rightsquigarrow \epsilon''$) the share construct is true since the field value old contains a true value (the one coming from ϵ'), and
- the same holds for further following events ϵ by inductive arguments.

In field calculus without share, such optimal communication speed can be achieved only through unbounded recursion, as argued in [ABDV18] and reviewed above in Section 2.5.

As a further example of successful application of Rewriting 3, consider the following routine where maxHoodPlusSelf is a built-in function returning the maximum value in the range of a numeric neighboring value.

```
def sharedcounter1() {
    rep (0) { (old) => max(maxHoodPlusSelf(nbr{old}), rep(0){(c)=>c+1}) }
}
```

This function computes a local counter through rep(0){(c)=>c+1} and then uses it to compute the maximum number of rounds a device in the network has performed (even though information about the number of rounds for other devices propagates at reduced speed). If we rewrite this function by eliminating the first rep through Rewriting 3, we obtain:

```
def sharedcounter2() {
    share (0) { (old) => max(maxHoodPlusSelf(old), rep(0){(c)=>c+1}) }
}
```

where information about the number of rounds for other devices is propagated to neighbors at the full multi-path speed allowed by **share**. It is worth observing that eliminating the remaining **rep** by further applying Rewriting 3 would produce the same result of applying Rewriting 1, i.e:

```
def sharedcounter() {
    share (0) { (old) => max(maxHoodPlusSelf(old), share(0){(c)=>localHood(c)+1}) }
}
```

and therefore would not affect the information propagation speed.

The average improvement in communication speed of a routine being converted from the usage of rep+nbr to share according to Rewriting 3 can also be statistically estimated, depending on the communication pattern used by the routine.

An algorithm follows a *single-path* communication pattern if its outcome in an event depends essentially on the value of a single selected neighbor: prototypical examples of such algorithms are distance estimations [ADV17, ADV18, ACDV17], which are computed out of the value of the single neighbor on the optimal path to the source. In this case, letting T be the average interval between subsequent rounds, the expected communication delay of an hop is T/2 with **share** (since it can randomly vary from 0 to T) and T/2 + T = 3/2T with **rep** + **nbr** (since a full additional round T is wasted in this case). Thus, the usage of **share** allows for an expected three-fold improvement in communication speed for these algorithms.

An algorithm follows a *multi-path* communication pattern if its outcome in an event is obtained from the values of all neighbors: prototypical examples of such algorithms are data collections [ABDV19], especially when they are idempotent (e.g. minimums or maximums). In this case, the existence of a single communication path $\epsilon_0 \sim \ldots \sim \epsilon$ is sufficient for the

value in ϵ_0 to be taken into account in ϵ . Even though the delay of any one of such paths follows the same distribution as for single-path algorithms (0 to T per step with **share**, T to 2T per step with **rep** + **nbr**), the overall delay is *minimized* among each existing path. It follows that for sufficiently large numbers of paths, the delay is closer to the minimum of a single hop (0 with **share**, T with **rep** + **nbr**) resulting in an even larger improvement.

3.4. Limitations of the Automatic Rewriting. In the previous section, we showed how the non-equivalent rewriting of rep + nbr statements into share statements is able to improve the performance of algorithms, both in the specific case of the ever and sharedcounter functions, and statistically for the communication speed of general algorithms. However, this procedure may *not* work for all functions: for example, consider the following routine

```
def fragilesharedcounter() {
  rep (0) { (old) => maxHoodPlusSelf(nbr{old})+1 }
}
```

that, if the scheduling of computation rounds is sufficiently regular across the network, is able to approximate the maximum number of rounds a device in the network has performed (even though information about the number of rounds for other devices propagates at reduced speed). If we rewrite this function through Rewriting 3, we obtain:

```
def fragilesharedcounter1() {
   share (0) { (old) => maxHoodPlusSelf(old)+1 }
}
```

which does *not* approximate the same quantity. Instead, it computes the maximum length of a path of messages reaching the current event, which may be unboundedly higher than round counts in case of dense networks.

In fact, the fragile shared counter function is a paradigmatic example of rewriting failure: it is a time-based function, whose results are strongly altered by removing the one-round wait generated by rep + nbr. Another class of programs for which the rewriting fails is that of functions with weakly defined behavior, usually based on heuristics, for which the increase in responsiveness may increase the fluctuations in results (or even prevent stabilisation to a meaningful value).

3.5. The share Construct Preserves Self-stabilisation. In this section, we prove that the automatic rewriting is able to improve an important class of functions with strongly defined behavior: the *self-stabilising fragment* of field calculus identified in $[VAB^+18]$. Functions complying to the syntactic and semantic restrictions imposed by this fragment are guaranteed to be *self-stabilising*, that is, whenever the function inputs and network structure stop changing, the output values will eventually converge to a value which only depends on the limit inputs and network structure (and not on what happened before the convergence of the network). This property captures the ability of a function to react to input changes, self-adjusting to the new correct value, and is thus a commonly used notion for strongly defining the behavior of a distributed function.

For example, function ever is not self-stabilising: if the inputs stabilise to being false everywhere, the function output could still be true if some past input was indeed true. As a positive example, the following function is self-stabilising, and computes the hop-count distance from the closest device where source is true.

s	::=	$\texttt{x} \ \big \ \texttt{v} \ \big \ \texttt{f}(\overline{\texttt{s}}) \ \big \ \texttt{if}(\texttt{s})\{\texttt{s}\} \{\texttt{s}\} \ \big \ \texttt{nbr}\{\texttt{s}\}$	self-stabilising expression with ${\tt rep}$
		$\Big \operatorname{rep}(e)\{(x) \Longrightarrow f^{C}(\operatorname{nbr}\{x\},\operatorname{nbr}\{s\},\overline{e})\}$	
		$ rep(e)\{(x) \Rightarrow f(mux(nbrlt(s), nbr\{x\}, s), \overline{s})\}$	
		$\Big \ \texttt{rep}(\texttt{e})\{(\texttt{x}) \texttt{=>} \texttt{f}^{R}(\texttt{minHoodLoc}(\texttt{f}^{MP}(\texttt{nbr}\{\texttt{x}\},\overline{\texttt{s}})$	$(\mathbf{s}), \mathbf{x}, \overline{\mathbf{e}}) \}$
s	::=	$x \ \big \ v \ \big \ f(\overline{s}) \ \big \ if(s) \{s\} \{s\} \ \big \ nbr\{s\}$	self-stabilising expression with ${\tt share}$
		$\Big \ \mathtt{share}(\mathtt{e})\{(\mathtt{x}) \texttt{=>} \mathtt{f}^{C}(\mathtt{x},\mathtt{nbr}\{\mathtt{s}\},\overline{\mathtt{e}})\}$	
		$\Big \ \mathtt{share}(\mathtt{e})\{(\mathtt{x}) \texttt{=>} \mathtt{f}(\mathtt{mux}(\mathtt{nbrlt}(\mathtt{s}),\mathtt{x},\mathtt{s}),\overline{\mathtt{s}})\}$	
	$\big \texttt{ share}(\texttt{e})\{(\texttt{x}) \texttt{=} \texttt{f}^{R}(\texttt{minHoodLoc}(\texttt{f}^{MP}(\texttt{x},\overline{\texttt{s}}),\texttt{s}),\texttt{localHood}(\texttt{x}),\overline{\texttt{e}})\}$		

Figure 7: Syntax of the self-stabilising fragment of field calculus introduced in [VAB⁺18], together with its translation through Rewriting 3. Self-stabilising expressions s occurring inside rep and share statements cannot contain free occurrences of the share-bound variable x.

```
def hopcount(source) {
   share (infinity) { (old) => mux(source, 0, minHood(old)+1) }
}
```

Here, minHood computes the minimum in the range of a numeric neighboring value (excluding the current device), while mux (multiplexer) selects between its second and third argument according to the value of the first (similarly as if, but evaluating all arguments).

A rewriting of the self-stabilising fragment with share is given in Figure 7, defining a class s of self-stabilising expressions, which may be:

- any expression not containing a **share** or **rep** construct, comprising built-in functions;
- three special forms of **share**-constructs, called *converging*, *acyclic* and *minimising* pattern (respectively), defined by restricting both the syntax and the semantic of relevant functional parameters.

We recall here a brief description of the patterns: for a more detailed presentation, the interested reader may refer to [VAB⁺18]. The semantic restrictions on functions are the following.

- **Converging (C):** A function $\mathbf{f}(\phi, \psi, \overline{\mathbf{v}})$ is said converging iff, for every device δ , its return value is closer to $\psi(\delta)$ than the maximal distance of ϕ to ψ .
- Monotonic non-decreasing (M): a stateless⁵ function $f(x, \overline{x})$ with arguments of local type is M iff whenever $\ell_1 \leq \ell_2$, also $f(\ell_1, \overline{\ell}) \leq f(\ell_2, \overline{\ell})$.
- **Progressive** (P): a stateless function $f(x, \overline{x})$ with arguments of local type is P iff $f(\ell, \overline{\ell}) > \ell$ or $f(\ell, \overline{\ell}) = \top$ (where \top denotes the unique maximal element of the relevant type).
- **Raising (R):** a function $f(\ell_1, \ell_2, \overline{v})$ is raising with respect to total partial orders <, \lhd iff: (i) $f(\ell, \ell, \overline{v}) = \ell$; (ii) $f(\ell_1, \ell_2, \overline{v}) \ge \min(\ell_1, \ell_2)$; (iii) either $f(\ell_1, \ell_2, \overline{v}) \rhd \ell_2$ or $f(\ell_1, \ell_2, \overline{v}) = \ell_1$.

Hence, the three patterns can be described as follows.

⁵A function $f(\bar{\mathbf{x}})$ is *stateless* iff its outputs depend only on its inputs and not on other external factors.

- **Converging:** In this pattern, variable \mathbf{x} is repeatedly updated through function \mathbf{f}^{C} and a self-stabilising value \mathbf{s} . The function \mathbf{f}^{C} may also have additional (not necessarily self-stabilising) inputs $\overline{\mathbf{e}}$. If the range of the metric granting convergence of \mathbf{f}^{C} is a well-founded set⁶ of real numbers, the pattern self-stabilises since it gradually approaches the value given by \mathbf{s} .
- Acyclic: In this pattern, the neighborhood's values for x are first filtered through a self-stabilising partially ordered "potential", keeping only values held in devices with lower potential (thus in particular discarding the device's own value of x). This is accomplished by the built-in function nbrlt, which returns a field of booleans selecting the neighbors with lower argument values, and could be defined as def nbrlt(x) {nbr{x} < x}.

The filtered values are then combined by a function f (possibly together with other values obtained from self-stabilising expressions) to form the new value for x. No semantic restrictions are posed in this pattern, and intuitively it self-stabilises since there are no cyclic dependencies between devices.

Minimising: In this pattern, the neighborhood's values for **x** are first increased by a monotonic progressive function \mathbf{f}^{MP} (possibly depending also on other self-stabilising inputs). As specified above, \mathbf{f}^{MP} needs to operate on local values: in this pattern it is therefore implicitly promoted to operate (pointwise) on fields.

Afterwards, the minimum among those values and a local self-stabilising value is then selected by function minHoodLoc(ϕ, ℓ) (which selects the "minimum" in $\phi[\delta \mapsto \ell]$). Finally, this minimum is fed to the *raising* function \mathbf{f}^{R} together with the old value for \mathbf{x} (and possibly any other inputs $\overline{\mathbf{e}}$), obtaining a result that is higher than at least one of the two parameters. We assume that the second partial order \triangleleft is *noetherian*,⁷ so that the raising function is required to eventually conform to the given minimum.

Intuitively, this pattern self-stabilises since it computes the minimum among the local values ℓ after being increased by \mathbf{f}^{MP} along every possible path (and the effect of the raising function can be proved to be negligible).

For expressions in the self-stabilising fragment, we can prove that the non-equivalent rewriting preserves the limit behavior, and thus may be safely applied in most cases.

Theorem 3.4. Assume that every built-in operator is self-stabilising. Then closed expressions s as in Figure 7 self-stabilise to the same limit for rep + nbr as their rewritings with share, the latter with a tighter bound on the number of full rounds of execution⁸ of a network needed before stabilisation.

Proof. See Appendix A.

4. Application and Empirical Validation

Having developed the **share** construct and shown that it should be able to significantly improve the performance of field calculus programs, we have also applied this development by extending the Protelis [PVB15] implementation of field calculus to support **share** (the

 $^{^{6}}$ An ordered set is *well-founded* iff it does not contain any infinite descending chain.

⁷A partial order is *noetherian* iff it does not contain any infinite ascending chains.

⁸A full round of execution is a sequence firings encompassing each device at least once.

implementation is a simple addition of another keyword and accompanying implementation code following the semantics expressed above). We have further upgraded every function in the protelis-lang library [FPBV17] with an applicable rep/nbr combination to use the share construct instead, thereby also improving every program that makes use of these libraries of resilient functions. The official Protelis distribution includes these changes to the language and the library into the main distribution, starting with version 11.0.0. To validate the efficacy of both our analysis and its applied implementation, we empirically validate the improvements in performance for a number of these upgraded functions in simulation.

4.1. Evaluation Setup. We experimentally validate the improvements of the share construct through two simulation examples. In both, we deploy a number of mobile devices, computing rounds asynchronously at a frequency of 1 ± 0.1 Hz, and communicating within a range of 75 meters. All aggregate programs have been written in Protelis [PVB15] and simulations performed in the Alchemist environment [PMV13]. All the results reported in this paper are the average of 200 simulations with different seeds, which lead to different initial device locations, different waypoint generation, and different round frequency. Data generated by the simulator has been processed with Xarray [HH17] and matplotlib [Hun07]. For the sake of brevity, we do not report the actual code in this paper; however, to guarantee complete reproducibility, the execution of the experiments has been entirely automated, and all the resources have been made publicly available along with instructions.⁹

In the first scenario, we position 2000 mobile devices into a corridor room with sides of, respectively, 200m and 2000m. All but two of the devices are free to move within the corridor randomly, while the remaining two are "sources" are fixed. We experiment with different locations for the latter, ranging from the opposite ends of the corridor to a distance of 100m. At every point of time, only one of the two sources is active, switching at 80 seconds and 200 seconds (i.e., the active one gets disabled, the disabled one is re-enabled). Devices are programmed to compute a field yielding everywhere the farthest distance from any device to the current active source. In order to do so, they execute the following commonly used coordination algorithms:

- they compute a potential field measuring the distance from the active source through BIS [ADV18] (bisGradient routine in protelis:coord:spreading);
- (2) they accumulate the maximum distance value descending the potential towards the source, through Parametric Weighted Multi-Path C [ABDV19] (an optimized version of C in protelis:coord:accumulation);
- (3) they broadcast the information along the potential, from the source to every other device in the system (an optimized version of the broadcast algorithm available in protelis:coord:spreading, which tags values from the source with a timestamp and propagates them by selecting more recent values).

The choice of the algorithms to be used in validation revealed to be critical. The usage of **share** is able to directly improve the performance of algorithms with solid theoretical guarantees; however, it may also exacerbate errors and instabilities for more ad-hoc algorithms, by allowing them to propagate quicker and more freely, preventing (or slowing down) the

⁹ Experiments are separated in two blocks, available on two separate repositories:

https://bitbucket.org/danysk/experiment-2019-coordination-aggregate-share/

https://github.com/DanySK/Experiment-2019-LMCS-Share



Figure 8: Performance in the corridor scenario, for both individual algorithms (top) and the composite computation (bottom). Vertical axis is linear in [0, 1] and logarithmic above. Charts on the left column show distance error, while the right column shows time error. The versions of the algorithms implemented with share (warm colors) produce significantly less error and converge significantly faster in case of large disruptions than with rep (cold colors).

stabilization of the algorithm result whenever the network configuration and input is not constant. Of the set of available algorithms for spreading and collecting data, we thus selected variants with smoother recovery from perturbation: optimal single-path distance estimation (BIS gradient [ADV18]), optimal multi-path broadcast [VAB⁺18], and the latest version of data collection (parametric weighted multi-path [ABDV19], fine-tuning the weight function).

We are interested in measuring the error of each step (namely, in distance vs. the true values), together with the lag through which these values were generated (namely, by propagating a time-stamp together with values, and computing the difference with the current time). Moreover, we want to inspect how the improvements introduced by **share** accumulate across the composition of algorithms. To do so, we measure the error in two conditions: (i) composite behavior, in which each step is fed the result computed by the previous step, and (ii) individual behavior, in which each step is fed an ideal result for the previous step, as provided by an oracle.

Figure 8 shows the results from this scenario. Observing the behavior of the individual computations, it is immediately clear how the **share**-based version of the algorithm provides faster recovery from network input discontinuities and lower errors at the limit. These effects are exacerbated when multiple algorithms are composed to build aggregate applications. The only counterexample is the limit of distance estimations, for which **rep** is marginally better, with a relative error less than 1% lower than that of **share**.

Moreover, notice that the collection algorithm with **rep** was not able to recover from changes at all, as shown by the linearly increasing delay in time (and the absence of spikes



Figure 9: Performance in the corridor scenario, showing on top the packet lag between the two fixed devices for the scenario in which they are at opposite ends of the corridor, and on bottom how the average packet lag changes with the distance between such devices. Broadcast data is on the left, accumulation on the right. Thinner lines depict mean \pm standard deviation. Darker lines depict "stacked" computations, namely, they use respectively rep-based or share-based algorithms to compute distances; lighter lines depict "single" computations, where distances are provided by an oracle. The versions of the algorithms implemented with share (warm colors) stabilize faster, and once stabilized they provide much lower network lags. The effect stacks when multiple algorithms are used together, as shown by the chart on packet delay in accumulation (top right): the collection algorithm using the distance computed with **rep** requires a longer time for stabilization, after which provides the same performance (in terms of lag) of the version relying on an oracle. Bottom charts show how both implementations scale linearly with the distance between devices (hence, for a network, linearly in its diameter); however, for **rep**-based algorithms scaling is noticeably worse.

in distance error). The known weakness of multi-path collection strategies, that is, failing to react to changes due to the creation of information loops, proved to be much more relevant and invalidating with **rep** than with **share**.

Further details on the improvements introduced by **share** are depicted in Figure 9, which shows both the lag between two selected devices and how such lag is influenced by the distance between them. Algorithms implemented on **share** provide, as expected, significantly lower network lags, and the effect is more pronounced as the distance between nodes increases: in fact, even though network lags expectedly scale linearly in both cases, **rep**-based versions accumulate lag much more quickly.

In the second example, we deploy 500 devices in a city center, and let them move as though being carried by pedestrians, moving at walking speed $(1.4\frac{m}{s})$ towards random waypoints along roads open to pedestrian traffic (using map data from OpenStreetMaps



Figure 10: Snapshots of the Voronoi partitioning scenario using share (left) or rep (right). Colored dots are simulated devices, with each region having a different color. Faster communication with share leads to a higher accuracy in distance estimation, allowing the share implementation to perform a better division into regions and preventing regions from expanding beyond their limits: note the mixing of colors on the right.

[HW08]). In this scenario, devices must self-organize service management regions with a radius of at most 200 meters, creating a Voronoi partition as shown in Figure 10 (functions S and voronoiPatitioningWithMetric from protelis:coord:sparsechoice). We evaluate performance by measuring the number of partitions generated by the algorithm, and the average and maximum node distance error, where the error for a node n measures how far a node is beyond of the maximum boundary for its cluster. This is computed as $\epsilon_n = \max(0, d(n, l_n) - r)$, where d computes the distance between two devices, l_n is the leader for the cluster n belongs to, and r is the maximum allowed radius of the cluster.

Figure 11 shows the results from this scenario, which also confirm the benefits of faster communication with **share**. The algorithm implemented with **share** has much lower error, mainly due to faster convergence of the distance estimates, and consequent higher accuracy in measuring the distance from the partition leader. Simultaneously, it creates a marginally lower number of partitions, by reducing the amount of occasional single-device regions which arise during convergence and re-organization.

5. CONCLUSION AND FUTURE WORK

We have introduced a novel **share** construct whose introduction allows a significant acceleration of field calculus programs. We have also made this construct available for use in applications though an extension of the Protelis field calculus implementation and its



Figure 11: Performance in the Voronoi partition scenario: error in distance on the left, leaders count with time on the right. Vertical axis is linear in [0, 0.1] and logarithmic elsewhere. The version implemented with **share** has much lower error: the mean error is negligible, and the most incorrect value, after an initial convergence phase, is close to two orders of magnitude lower than with **rep**, as faster communication leads to more accurate distance estimates. The leader count shows that the systems create a comparable number of partitions, with the **share**-based featuring faster convergence to a marginally lower number due to increased consistency in partitioning.

accompanying libraries, and have empirically validated the expected improvements in performance through experiments in simulation.

In future work, we plan to study for which algorithms the usage of share may lead to increased instability, thus fine-tuning the choice of rep and nbr over share in the Protelis library. Furthermore, we intend to fully analyze the consequences of share for improvement of space-time universality [ABDV18], self-adaption [BVPD17], and variants of the semantics [ADVC16] of the field calculus. It also appears likely that the field calculus can be simplified by the elimination of both rep and nbr by finding a mapping by which share can also be used to implement any usage of nbr. Finally, we believe that the improvements in performance will also have positive consequences for nearly all current and future applications that are making use of the field calculus and its implementations and derivatives.

Acknowledgements. We thank the anonymous COORDINATION 2019 referees for their comments and suggestions on improving the presentation.

References

- [ABD⁺19] Giorgio Audrito, Jacob Beal, Ferruccio Damiani, Danilo Pianini, and Mirko Viroli. The share operator for field-based coordination. In *Coordination Models and Languages*, volume 11533 of *Lecture Notes in Computer Science*, pages 54–71. Springer, 2019.
- [ABDV18] Giorgio Audrito, Jacob Beal, Ferruccio Damiani, and Mirko Viroli. Space-time universality of field calculus. In *Coordination Models and Languages*, volume 10852 of *Lecture Notes in Computer Science*, pages 1–20. Springer, 2018.
- [ABDV19] Giorgio Audrito, Sergio Bergamini, Ferruccio Damiani, and Mirko Viroli. Effective collective summarisation of distributed data in mobile multi-agent systems. In *International Conference* on Autonomous Agents and Multiagent Systems (AAMAS). ACM, 2019.
- [ACDV17] Giorgio Audrito, Roberto Casadei, Ferruccio Damiani, and Mirko Viroli. Compositional blocks for optimal self-healing gradients. In 11th International Conference on Self-Adaptive and Self-Organizing Systems (SASO 2017), pages 91–100. IEEE, 2017.

- [ADV17] Giorgio Audrito, Ferruccio Damiani, and Mirko Viroli. Optimally-self-healing distributed gradient structures through bounded information speed. In *Coordination Models and Languages*, volume 10319 of *LNCS*, pages 59–77. Springer, 2017.
- [ADV18] Giorgio Audrito, Ferruccio Damiani, and Mirko Viroli. Optimal single-path information propagation in gradient-based algorithms. *Science of Computer Programming*, 166:146–166, 2018.
- [ADVC16] Giorgio Audrito, Ferruccio Damiani, Mirko Viroli, and Roberto Casadei. Run-time management of computation domains in field calculus. In 1st International Workshops on Foundations and Applications of Self* Systems (FAS*W), pages 192–197. IEEE, 2016.
- [ARGL⁺07] Michael P. Ashley-Rollman, Seth Copen Goldstein, Peter Lee, Todd C. Mowry, and Padmanabhan Pillai. Meld: A declarative approach to programming ensembles. In *IEEE International Conference on Intelligent Robots and Systems (IROS '07)*, pages 2794–2800, 2007.
- [AVD⁺19] Giorgio Audrito, Mirko Viroli, Ferruccio Damiani, Danilo Pianini, and Jacob Beal. A higherorder calculus of computational fields. ACM Transactions on Computational Logic (TOCL), 20(1):5:1–5:55, 2019.
- [BB06] Jacob Beal and Jonathan Bachrach. Infrastructure for engineered emergence in sensor/actuator networks. *IEEE Intelligent Systems*, 21:10–19, March/April 2006.
- [BDU⁺13] Jacob Beal, Stefan Dulman, Kyle Usbeck, Mirko Viroli, and Nikolaus Correll. Organizing the aggregate: Languages for spatial computing. In Formal and Practical Aspects of Domain-Specific Languages: Recent Developments, chapter 16, pages 436–501. IGI Global, 2013.
- [BPV15] Jacob Beal, Danilo Pianini, and Mirko Viroli. Aggregate programming for the Internet of Things. *IEEE Computer*, 48(9), 2015.
- [But02] William Butera. Programming a Paintable Computer. PhD thesis, MIT, Cambridge, USA, 2002.
- [BVPD17] Jacob Beal, Mirko Viroli, Danilo Pianini, and Ferruccio Damiani. Self-adaptation to device distribution in the Internet of Things. ACM Transactions on Autonomous and Adaptive Systems (TAAS), 12(3):12:1–12:29, 2017.
- [CGG⁺05] Carlo Curino, Matteo Giani, Marco Giorgetta, Alessandro Giusti, Amy L. Murphy, and Gian Pietro Picco. Mobile data collection in sensor networks: The tinylime middleware. *El-sevier Pervasive and Mobile Computing Journal*, 4:446–469, 2005.
- [Chu32] Alonzo Church. A set of postulates for the foundation of logic. Annals of Mathematics, 33(2):346–366, 1932.
- [CN03] Lauren Clement and Radhika Nagpal. Self-assembly and self-repairing topologies. In Workshop on Adaptability in Multi-Agent Systems, RoboCup Australian Open, 2003.
- [Coo99] Daniel Coore. Botanical Computing: A Developmental Approach to Generating Inter connect Topologies on an Amorphous Computer. PhD thesis, MIT, Cambridge, MA, USA, 1999.
- [DG08] Jeffrey Dean and Sanjay Ghemawat. Mapreduce: simplified data processing on large clusters. Communications of the ACM, 51(1):107–113, 2008.
- [FPBV17] Matteo Francia, Danilo Pianini, Jacob Beal, and Mirko Viroli. Towards a foundational api for resilient distributed systems design. In 2017 IEEE 2nd International Workshops on Foundations and Applications of Self* Systems (FAS* W), pages 27–32. IEEE, 2017.
- [GGG05] Ramakrishna Gummadi, Omprakash Gnawali, and Ramesh Govindan. Macro-programming wireless sensor networks using kairos. In *Distributed Computing in Sensor Systems (DCOSS)*, pages 126–140, 2005.
- [GGMP02] Jean-Louis Giavitto, Christophe Godin, Olivier Michel, and Przemysław Prusinkiewicz. Computational models for integrative and developmental biology. Technical Report 72-2002, U. d'Evry, LaMI, 2002.
- [GMCS05] Jean-Louis Giavitto, Olivier Michel, Julien Cohen, and Antoine Spicher. Computations in space and space in computations. In Unconventional Programming Paradigms, volume 3566 of Lecture Notes in Computer Science, pages 137–152. Springer, Berlin, 2005.
- [HH17] S. Hoyer and J. Hamman. xarray: N-D labeled arrays and datasets in Python. Journal of Open Research Software, 5(1), 2017.
- [Hun07] J. D. Hunter. Matplotlib: A 2d graphics environment. Computing In Science & Engineering, 9(3):90–95, 2007.
- [HW08] M. Haklay and P. Weber. OpenStreetMap: User-generated street maps. IEEE Pervasive Computing, 7(4):12–18, oct 2008.

- [IPW01] Atsushi Igarashi, Benjamin C. Pierce, and Philip Wadler. Featherweight Java: A minimal core calculus for Java and GJ. ACM Transactions on Programming Languages and Systems, 23(3), 2001.
- [Kon03] Attila Kondacs. Biologically-inspired self-assembly of 2d shapes, using global-to-local compilation. In International Joint Conference on Artificial Intelligence (IJCAI), pages 633–638. Morgan Kaufmann Publishers Inc., 2003.
- [LMMD88] C. Lasser, J.P. Massar, J. Miney, and L. Dayton. Starlisp Reference Manual. Thinking Machines Corporation, 1988.
- [MFHH02] Samuel Madden, Michael J. Franklin, Joseph M. Hellerstein, and Wei Hong. TAG: A Tiny AGgregation Service for Ad-hoc Sensor Networks. SIGOPS Oper. Syst. Rev., 36:131–146, 2002.
- [MZ09] Marco Mamei and Franco Zambonelli. Programming pervasive and mobile computing applications: The tota approach. ACM Transactions on Software Engineering Methodologies (TOSEM), 18(4):1–56, 2009.
- [Nag01] Radhika Nagpal. Programmable Self-Assembly: Constructing Global Shape using Biologicallyinspired Local Interactions and Origami Mathematics. PhD thesis, MIT, Cambridge, MA, USA, 2001.
- [NW04] Ryan Newton and Matt Welsh. Region streams: Functional macroprogramming for sensor networks. In Workshop on Data Management for Sensor Networks, DMSN '04, pages 78–87. ACM, 2004.
- [PMV13] Danilo Pianini, Sara Montagna, and Mirko Viroli. Chemical-oriented simulation of computational systems with ALCHEMIST. J. Simulation, 7(3):202–215, 2013.
- [PVB15] Danilo Pianini, Mirko Viroli, and Jacob Beal. Protelis: Practical aggregate programming. In ACM Symposium on Applied Computing 2015, pages 1846–1853, April 2015.
- [VAB⁺18] Mirko Viroli, Giorgio Audrito, Jacob Beal, Ferruccio Damiani, and Danilo Pianini. Engineering resilient collective adaptive systems by self-stabilisation. ACM Transactions on Modelling and Computer Simulation (TOMACS), 28(2):16:1–16:28, 2018.
- [Val90] Leslie G Valiant. A bridging model for parallel computation. Communications of the ACM, 33(8):103–111, 1990.
- [VBD⁺18] Mirko Viroli, Jacob Beal, Ferruccio Damiani, Giorgio Audrito, Roberto Casadei, and Danilo Pianini. From field-based coordination to aggregate computing. In *Coordination Models and Languages*, volume 10852 of *Lecture Notes in Computer Science*, pages 252–279. Springer, 2018.
- [VPM⁺15] Mirko Viroli, Danilo Pianini, Sara Montagna, Graeme Stevenson, and Franco Zambonelli. A coordination model of pervasive service ecosystems. Science of Computer Programming, 110:3 - 22, 2015.
- [WSBC04] Kamin Whitehouse, Cory Sharp, Eric Brewer, and David Culler. Hood: a neighborhood abstraction for sensor networks. In *Proceedings of the 2nd international conference on Mobile* systems, applications, and services. ACM Press, 2004.
- [Yam07] Daniel Yamins. A Theory of Local-to-Global Algorithms for One-Dimensional Spatial Multi-Agent Systems. PhD thesis, Harvard, Cambridge, MA, USA, 2007.
- [YG02] Yong Yao and Johannes Gehrke. The cougar approach to in-network query processing in sensor networks. SIGMOD Record, 31:9–18, 2002.

APPENDIX A. PROOF OF SELF-STABILISATION

In this section, we prove Theorem 3.4. First, we prove the result for the minimising pattern (Lemma A.1), since it is technically more involved than the proof for the remainder of the fragment. We then prove a stronger form of the desired result (Lemma A.2) more suited for inductive reasoning, which in turn implies Theorem 3.4.

Given a self-stabilising expression \mathbf{s} , we denote with $[\![\mathbf{s}]\!] = \Phi = \overline{\delta} \mapsto \overline{\mathbf{v}}$ the self-stabilising limit value of this expression in a given network. This limit is attained for every *fair* evolution of a network, that is, an infinite sequence of firings $N_0 \xrightarrow{\delta_0} N_1 \xrightarrow{\delta_1} \ldots$ encompassing every device in the network infinitely often (in other words, containing an infinite number

of full rounds of execution). Let:

$$\begin{split} \mathbf{s}_{\min}^{r} &= \mathtt{rep}(\mathbf{e})\{(\mathbf{x}) \texttt{=>} \mathtt{f}^{\mathsf{R}}(\mathtt{minHoodLoc}(\mathtt{f}^{\mathsf{MP}}(\mathtt{nbr}\{\mathbf{x}\}, \overline{\mathbf{s}}^{r}), \mathbf{x}^{r}), \mathbf{x}, \overline{\mathbf{e}})\}\\ \mathbf{s}_{\min}^{s} &= \mathtt{share}(\mathbf{e})\{(\mathbf{x}) \texttt{=>} \mathtt{f}^{\mathsf{R}}(\mathtt{minHoodLoc}(\mathtt{f}^{\mathsf{MP}}(\mathbf{x}, \overline{\mathbf{s}}^{s}), \mathbf{s}^{s}), \mathtt{localHood}(\mathbf{x}), \overline{\mathbf{e}})\} \end{split}$$

be corresponding minimising patterns such that $[\![\overline{\mathbf{s}}^r]\!] = [\![\overline{\mathbf{s}}^s]\!] = \overline{\Phi}, [\![\mathbf{s}^r]\!] = [\![\mathbf{s}^s]\!] = \Phi$. Let $P = \overline{\delta}$ be a path in the network (a sequence of pairwise connected devices), and define its weight as the result of picking the eventual value $\ell_1 = \Phi(\delta_1)$ of \mathbf{s}^r in the first device δ_1 , and repeatedly passing it to subsequent devices through the monotonic progressive function, so that $\ell_{i+1} = \mathbf{f}^{\mathsf{MP}}(\ell_i, \overline{\mathbf{v}})$ where $\overline{\mathbf{v}}$ is the result of projecting fields in $\overline{\Phi}(\delta_{i+1})$ to their δ_i component (leaving local values untouched). Notice that the weight is well-defined since function \mathbf{f}^{MP} is required to be stateless. Finally, let Φ_{out} be such that $\Phi_{\mathsf{out}}(\delta) = \ell_{\delta}$ is the minimum weight for a path P ending in δ .

Lemma A.1. Let \mathbf{s}_{min}^r , \mathbf{s}_{min}^s be corresponding minimising patterns. Then they both selfstabilise to Φ_{out} , with a bound on the number of full rounds of execution (after stabilising of sub-expressions) which is lower for \mathbf{s}_{min}^s than for \mathbf{s}_{min}^r .

Proof. Let ℓ_{δ} be the minimal weight for a path P ending in δ , and let $\delta^0, \delta^1, \ldots$ be the list of all devices δ ordered by increasing ℓ_{δ} . Notice that the path P of minimal weight $\ell_{\delta i}$ for device i can only pass through nodes such that $\ell_{\delta j} \leq \ell_{\delta i}$ (thus s.t. j < i). In fact, whenever a path P contains a node j the weight of its prefix until j is at least $\ell_{\delta j}$; thus any longer prefix has weight strictly greater than $\ell_{\delta j}$ since \mathbf{f}^{MP} is progressive.

Let $N_0 \xrightarrow{\delta_0} N_1 \xrightarrow{\delta_1} \ldots$ be a fair evolution¹⁰ and assume w.l.o.g. that subexpressions $\overline{\mathbf{s}}^r$, $\overline{\mathbf{s}}^s$, \mathbf{s}^r , \mathbf{s}^s have already self-stabilised to computational fields $\overline{\Phi}$, Φ (as in the definition of weight) in the initial state N_0 . We now prove by complete induction on *i* that expressions \mathbf{s}_{\min}^r , \mathbf{s}_{\min}^s stabilise to $\ell_{\delta i}$ in device δ^i after a certain number of steps t_i^r , t_i^s .

Assume that devices δ^j with j < i are all self-stabilised and their limit values are available to neighbours¹¹ from a certain number of steps t_{i-1}^r , t_{i-1}^s . Consider the evaluation of the expressions \mathbf{s}_{\min}^r , \mathbf{s}_{\min}^s in a device δ^k with $k \ge i$. Since the local argument ℓ of **minHoodLoc** is also the weight of the single-node path $P = \delta^k$, it has to be at least $\ell \ge \ell_{\delta k} \ge \ell_{\delta i}$. Similarly, the restriction ϕ' of the field argument ϕ of **minHoodLoc** to devices δ^j with j < i has to be at least $\phi' \ge \ell_{\delta k} \ge \ell_{\delta i}$ since it corresponds to weights of (not necessarily minimal) paths P ending in δ^k (obtained by extending a minimal path for a device δ^j with j < i with the additional node δ^k). Finally, the complementary restriction ϕ'' of ϕ to devices δ^j with $j \ge i$ is strictly greater than the minimum value for whole \mathbf{s}_{\min}^r , \mathbf{s}_{\min}^s expression among those devices (delayed by one round for $\mathbf{rep} + \mathbf{share}$), since \mathbf{f}^{MP} is progressive.

It follows that as long as the minimum value for the whole expressions among non-stable devices is lower than ℓ_{δ^i} , the result of the minHoodLoc subexpression is *strictly greater* than this minimum value. The same holds for the overall value, since it is obtained by combining the output of minHoodLoc with the previous value for x through the rising function \mathbf{f}^{R} , and a rising function has to be equal to the first argument (the minHoodLoc result strictly greater than the minimum), or \triangleright than the second. In the latter case, it also needs to be

¹⁰Notice that δ_0 is the first device firing while δ^0 is the device with minimal weight.

¹¹For rep + nbr, one additional full round of execution is needed between stabilisation on the limit being available; whereas for **share** the limit values are immediately available after stabilisation.

greater or equal to the first argument (again, strictly greater than the minimum) or strictly greater than the second argument¹² (not below the minimum value).

Thus, every full round of execution (two full rounds for rep + nbr, in order to allow value changes to be received) the minimum value among non-stable devices has to increase, until it eventually surpasses $\ell_{\delta i}$. From that point on, that minimum cannot drop below $\ell_{\delta i}$, and the output of minHoodLoc in δ^i stabilises to $\ell_{\delta i}$. In fact, if P is a path of minimum weight for δ^i , then either:

- P = δⁱ, so that ℓ_{δⁱ} is exactly the local argument of the minHoodLoc operator, hence also the output of it (since the field argument is greater than ℓ_{δⁱ}).
 P = Q, δⁱ where Q ends in δ^j with j < i. Since f^{MP} is monotonic non-decreasing,
- $P = Q, \delta^i$ where Q ends in δ^j with j < i. Since \mathbf{f}^{MP} is monotonic non-decreasing, the weight of Q', δ^i (where Q' is minimal for δ^j) is not greater than that of P; in other words, $P' = Q', \delta^i$ is also a path of minimum weight. It follows that $\phi(\delta^j)$ (where ϕ is the field argument of the minHoodLoc operator) is exactly ℓ_{δ^i} .

Since the order \triangleleft is noetherian, the rising function has to select its first argument infinitely often. In particular, on δ^i it will select the output of the minHoodLoc subexpression, which is ℓ_{δ^i} . From that point on, the minimising expression will have self-stabilised on device δ^i to ℓ_{δ^i} , concluding the inductive step and the proof.

Let Φ be a map from devices to values. We write $\mathbf{s}[\mathbf{x} := \Phi]$ to indicate an aggregate process in which each device is computing a possibly different substitution $\mathbf{s}[\mathbf{x} := \Phi(\delta)]$ of the same expression.

Lemma A.2. Assume that every built-in operator is self-stabilising. Let \mathbf{s}^r be an expression in the self-stabilising fragment of [VAB⁺18], \mathbf{s}^s its non-equivalent translation with share, and $\overline{\Phi}$ be a sequence of computational fields of the same length as the free variables $\overline{\mathbf{x}}$ occurring in \mathbf{s}^r , \mathbf{s}^s . Then $\mathbf{s}^r[\overline{\mathbf{x}} := \overline{\Phi}]$, $\mathbf{s}^s[\overline{\mathbf{x}} := \overline{\Phi}]$ self-stabilise to the same limit, and the second does so with a smaller bound on the number of full rounds of execution.

Proof. The proof proceeds by induction on the syntax of expressions and programs. The given expressions \mathbf{s}^r , \mathbf{s}^s could be:

- A variable \mathbf{x}_i , so that $\mathbf{s}^r[\overline{\mathbf{x}} := \overline{\Phi}] = \mathbf{s}^s[\overline{\mathbf{x}} := \overline{\Phi}] = \Phi_i$ are already self-stabilised and identical.
- A value v, so that $\mathbf{s}^r[\overline{\mathbf{x}} := \overline{\Phi}] = \mathbf{s}^s[\overline{\mathbf{x}} := \overline{\Phi}] = \mathbf{v}$ are already self-stabilised and identical.
- A functional application f^r(s̄^r), f^s(s̄^s). Fix an environment *Env*, in which all expressions s̄^r, s̄^s self-stabilise to Φ after a certain amount of full rounds of execution (lower for s̄^s by inductive hypothesis). After stabilisation of the arguments, if f^r = f^s = f is a built-in function then f(s̄^r), f(s̄^s) are already self-stabilised. Otherwise, f^r(s̄^r), f^s(s̄^r) evaluate to the same value of the expression body(f^r)[args(f^r) := Φ̄] (resp. with f^s) which are self-stabilising in a number of full rounds of executions lower for f^s by inductive hypothesis.
- A conditional $\mathbf{s}^r = i\mathbf{f}(\mathbf{s}_1^r)\{\mathbf{s}_2^r\}\{\mathbf{s}_3^r\}, \mathbf{s}^s = i\mathbf{f}(\mathbf{s}_1^s)\{\mathbf{s}_2^s\}\{\mathbf{s}_3^s\}$. Fix an environment *Env*, in which expressions \mathbf{s}_1^r , \mathbf{s}_1^s self-stabilise to Φ_{guard} (with fewer rounds for share by inductive hypothesis). Let Env_{true} be the sub-environment consisting of devices δ such that $\Phi_{guard}(\delta) = true$, and analogously Env_{false} . Assume that \mathbf{s}_2^r , \mathbf{s}_2^s self-stabilise to Φ_{true} in Env_{true} and \mathbf{s}_3^r , \mathbf{s}_3^s to Φ_{false} in Env_{false} (with fewer rounds

¹²It cannot be equal to the second argument, as it is \triangleright -greater than it.

for share). Since a conditional is computed in isolation in the above defined subenvironments, \mathbf{s}^r , \mathbf{s}^s self-stabilise to $\Phi = \Phi_{true} \cup \Phi_{false}$.

- A neighborhood field construction $nbr\{s^r\}$, $nbr\{s^s\}$. Fix an environment *Env*, in which expressions s^r , s^s self-stabilise to Φ after some rounds of computation (fewer for share). Then $nbr\{s^r\}$, $nbr\{s^s\}$ self-stabilises to the corresponding Φ' after one additional full round of execution, where $\Phi'(\delta)$ is Φ restricted to $\tau(\delta)$.
- A converging pattern $\mathbf{s}_c^r, \mathbf{s}_c^s$:

$$\begin{split} \mathbf{s}_{c}^{r} &= \mathtt{rep}(\mathbf{e})\{(\mathtt{x}) \Rightarrow \mathtt{f}^{\mathsf{C}}(\mathtt{nbr}\{\mathtt{x}\},\mathtt{nbr}\{\mathtt{s}^{r}\},\overline{\mathbf{e}})\}\\ \mathbf{s}_{c}^{s} &= \mathtt{share}(\mathbf{e})\{(\mathtt{x}) \Rightarrow \mathtt{f}^{\mathsf{C}}(\mathtt{x},\mathtt{nbr}\{\mathtt{s}^{s}\},\overline{\mathbf{e}})\} \end{split}$$

Fix an environment Env and a fair evolution of the network $N_0 \xrightarrow{\delta_0} N_1 \xrightarrow{\delta_1} \ldots$, and assume w.l.o.g. that \mathbf{s}^r , \mathbf{s}^s have already self-stabilised (the latter with fewer rounds of computation) to a same Φ ; we prove that \mathbf{s}^r_c , \mathbf{s}^s_c stabilise as well to the same Φ .

Given any index t, let d_t^r , d_t^s be the maximum distances $\mathbf{s}_c^r - \Phi(\delta_t^r)$, $\mathbf{s}_c^s - \Phi(\delta_t^s)$ respectively realised by devices δ_t^r , δ_t^s in N_t .¹³ Let $t_0^r = t_0^s = 0$ and t_{i+1}^s be the first firing of device $\delta_{t_i^s}^s$ after t_i^s , t_{i+1}^r be the first firing of device $\delta_{t_i^r}^r$ after one full round of execution after t_i^r . Since $\delta_{t_i^r}^r$ realises the maximum distance in the whole network $N_{t_i^r}$, no device firing of \mathbf{s}_c^r between t_i^r and t_{i+1}^r can assume a value more distant than $d_{t_i^r}^r$ without violating the converging property. Similarly, no device firing of \mathbf{s}_c^r remain constant in the whole interval from t_i^r to t_{i+1}^r (excluded), and respectively with d_t^s , δ_t^s in $[t_i^s, t_{i+1}^s]$.

Finally, in fire t_{i+1}^r device $\delta_{t_i^r}^r$ recomputes its value, necessarily obtaining a closer value to $\Phi(\delta_{t_i^r}^r)$ (by the converging property) thus forcing the overall maximal distance in the network to reduce: $d_{t_{i+1}^r}^r < d_{t_i^r}^r$ (respectively with $d_{t_i^s}^s$). Since the set of possible values is finite, so are the possible distances and eventually the maximal distances d_i^r , d_i^s will reach 0. In particular, d_i^s will reach 0 in a number of full rounds of execution lower than the number of possible distances V, while d_i^r will reach 0 in a number of full rounds of execution lower than 2V (after the stabilisation of \mathbf{s}^r , \mathbf{s}^s).

• An acyclic pattern \mathbf{s}_a^r , \mathbf{s}_a^s :

$$\mathbf{s}_{a}^{r} = \operatorname{rep}(\mathbf{e})\{(\mathbf{x}) \Rightarrow \mathbf{f}^{r}(\operatorname{mux}(\operatorname{nbrlt}(\mathbf{s}_{p}^{r}), \operatorname{nbr}\{\mathbf{x}\}, \mathbf{s}^{r}), \overline{\mathbf{s}}^{r})\}$$
$$\mathbf{s}_{a}^{s} = \operatorname{share}(\mathbf{e})\{(\mathbf{x}) \Rightarrow \mathbf{f}^{s}(\operatorname{mux}(\operatorname{nbrlt}(\mathbf{s}_{p}^{s}), \mathbf{x}, \mathbf{s}^{s}), \overline{\mathbf{s}}^{s})\}$$

Fix an environment Env and a fair evolution of the network $N_0 \xrightarrow{\delta_0} N_1 \xrightarrow{\delta_1} \ldots$, and assume w.l.o.g. that \mathbf{s}^r , \mathbf{s}^s have already self-stabilised (the latter with fewer rounds of computation) to a same Φ , and similarly for \mathbf{s}_p^r , \mathbf{s}_p^s with Φ_p and $\overline{\mathbf{s}}^r$, $\overline{\mathbf{s}}^s$ with $\overline{\Phi}$.

Let t be any fire of the device δ_0 of minimal potential $\Phi_p(\delta_0)$ in the network. Since $\Phi_p(\delta_0)$ is minimal, $nbrlt(\mathbf{s}_p^r)$ is false and $mux(nbrlt(\mathbf{s}_p^r), nbr\{\mathbf{x}\}, \mathbf{s}^r)$ reduces to \mathbf{s}^r and the whole \mathbf{s}_a^r to $\mathbf{f}^r(\mathbf{s}^r, \mathbf{\bar{s}}^r)$, which self-stabilises by inductive hypothesis after a certain round t_0^r (including one extra round for the stabilised value to be available to neighbours). The same applies to \mathbf{s}_a^s after t_0^s , with fewer full rounds of computation by inductive hypothesis.

¹³If multiple devices attain the same distance, we select the device with lower index.

Let now t be any fire of the device δ_1 of second minimal potential $\Phi_p(\delta_1)$ after t_0^r . Then $\max(\operatorname{nbrlt}(\mathbf{s}_p^r), \operatorname{nbr}\{\mathbf{x}\}, \mathbf{s}^r)$ in δ_1 only (possibly) depends on the value of the device of minimal potential, which is already self-stabilised and available to neighbours. Thus by inductive hypothesis \mathbf{s}_a^r self-stabilises also in δ_1 after some index $t_1^r \geq t_0^r$ (including one extra round for the value being available to neighbours). The same holds for \mathbf{s}_a^s after t_1^s with fewer rounds of computation. By repeating the same reasoning on all devices in order of increasing potential, we obtain final t_n^r , t_n^s after which all devices have self-stabilised.

• A minimising **rep**: this case is proved for closed expressions in Lemma A.1, and its generalisation to open expressions is straightforward.